0214841

12125

Reg. No. | | | | | | | | |

I Semester B.Sc. Degree Examination, August - 2021

MATHEMATICS

Mathematics - I

(CBCS Semester Scheme)

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates : Answer **all** questions.

I. Answer any **Five** questions. **(5×2=10)**

1) Define rank of a matrix A of order $m \times n$.

2) Show that the system of equations $x+2y+z=0$, $x-2z=0$, $2x+y-3z=0$ has only trivial solution.

3) Find the n[th] derivative of $\sin^2 x$.

4) If $u = x^2 y$, prove that $\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial^2 u}{\partial x \partial y}$.

5) Evaluate : $\int_0^{\frac{\pi}{2}} \sin^8 x \, dx$.

6) Evaluate : $\int_0^{\frac{\pi}{2}} \sin^6 x \cos^5 x \, dx$.

7) Find the value of k such that the spheres $x^2+y^2+z^2+4x+ky+2z+6=0$ and $x^2+y^2+z^2+2x-4y-2z+6=0$ are orthogonal.

8) Find the equation of the right circular cone whose vertex is $(1,2,3)$, axis is the y - axis and semi vertical angle is 30^0.

[P.T.O.

II. Answer any **Three** questions. (3×5=15)

9) Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 2 & 1 & -3 & -6 \\ 3 & -3 & 1 & 2 \end{bmatrix}$ by reducing to normal form.

10) Test the system of equations $x+2y-z=3$, $3x-y+2z=1$, $2x-2y+3z=2$ for consistency and solve if possible.

11) Find the eigen values and corresponding eigen vectors of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

12) State and prove Cayley - Hamilton Theorem.

13) By using Cayley - Hamilton Theorem, find the inverse of the matrix $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$.

III. Answer any **Three** questions. (3×5=15)

14) Find the n^th^ derivative of $\frac{4x}{(x+1)^2(x-1)}$.

15) If $y = e^{m\sin^{-1}x}$ prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+m^2)y_n = 0$.

16) If $Z = \sin(ax+y) + \cos(ax-y)$, prove that $\frac{\partial^2 Z}{\partial x^2} = a^2 \frac{\partial^2 Z}{\partial y^2}$.

17) State and prove Euler's Theorem for a homogeneous function of two variables.

18) If $x = r\cos\theta$, $y = r\sin\theta$, find $J = \frac{\partial(x,y)}{\partial(r,\theta)}$ and $J' = \frac{\partial(r,\theta)}{\partial(x,y)}$. Also verify $JJ' = 1$.

IV. Answer any **Two** questions. (2×5=10)

19) Obtain the reduction formula for $\int \cos^n x \, dx$.

20) Evaluate : $\int_0^\infty \frac{x^2}{(1+x^2)^3} dx$.

21) Using Leibnitz's rule, evaluate $\int_0^\infty e^{-x} \frac{\sin \alpha x}{x} dx$, where α is a parameter.

V. Answer any **Two** questions. **(2×5=10)**

22) Obtain the equation of the sphere which passes through the points $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and having its centre on the plane $3x-y+z=2$.

23) Find the equation of the right circular cone which passes through the point $(1,1,2)$, has its vertex at the origin and its axis is $\frac{x}{2}=\frac{-y}{4}=\frac{z}{3}$.

24) Find the equation of the right circular cylinder of radius 2 units and whose axis is $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z-2}{5}$.

VI. Answer any **Two** questions. **(2×5=10)**

25) Suppose in a stationery shop books, pens and pencils are sold with 3 books, 2 pens and 1 pencil cost Rs. 140, 2 books, 2 pens and 2 pencils cost Rs. 170, 1 book, 3 pens and 2 pencils cost Rs. 180. Find whether each item has a fixed price or each item has different prices or it is not possible to find the price of the items.

26) The population grows at the rate of 5% per year. How long does it take for the population to double?

27) A particle of mass 3 units is moving along the space curve defined by $\vec{r}=\left(4t^2+t^3\right)\hat{i}+5t\,\hat{j}+\left(t^3+2\right)\hat{k}$. Find

i) The momentum and

ii) Force acting on it at $t=2$.