DCMT201

Reg. No. [ ]

II Semester B.Sc. Degree Examination, Oct.- 2022

MATHEMATICS

Algebra - II and Calculus - II

(NEP-CORE)

Paper: II

Time : 2½ Hours **Maximum Marks : 60**

Instructions to Candidates:

Answer all the questions

I. Answer any Six questions. **(6×2=12)**

1. In the group of integers z an operation * is defined by a*b = a+b-3 $\forall a,b \in z$ find $(5*2)^{-1}$
2. Prove that every group of order 3 is abelian.
3. Prove that every subgroup of an abelian group is normal.
4. Show that $f:(G,+)\to(G',.)$ defined by $f(x)=e^x, \forall x\in G$ is a homomorphism.
5. Find the radius of curvature at any point (p,r) on the curve $r^3 = a^2p$
6. Find $\frac{ds}{dx}$ for the curve $y^2=4ax$.
7. Evaluate $\int_0^{\pi/2} \cos^5 x \, dx$
8. Find the asymptotes parallel to x - *axis* of the curve $y^3-x^2y+2y^2+4y+1=0$

II. Answer any THREE questions. **(3×4=12)**

9. Prove that set Q_1 of rational numbers other than 1 the binary operation a*b = a+b -ab is an abelian group.
10. Prove that a non-empty subset H of a group G is a subgroup of G if and only if $ab^{-1}\in H, \forall a,b\in H$.

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11. G is a group and $a \in G$ is an element of order n, prove that for any positive integer m, $a^m = e$ if and only if n is a divisor of m.

12. If H is a subgroup of a group G prove that there is a one to one correspondence between the set of all right cosets and the set of all left cosets of H in G.

13. State and prove lagrange's theorem in groups.

III. Answer any THREE questions. **(3×4=12)**

14. Prove that a subgroup H of a group G is normal if and only if $ghg^{-1} \in H, \forall g \in G, h \in H$

15. Prove that the product of two normal subgroups of a group is a sub group of a group.

16. Let (z,+) be the additive group of all integers and H = {1,-1} be the multiplicative group. Define $f(z)=\begin{cases} 1 & if\ z\ is\ even \\ -1 & if\ z\ is\ odd \end{cases}$

Show that f is a homomorphism.

17. If $f: G \to G'$ is a homomorphism then prove that

i) $f(e)=e'$, where e and e' are identity elements in G and G' respectively and

ii) $f(a^{-1})=(f(a))^{-1}, \forall a \in G$

18. State and prove fundamental theorem of homomorphism of group.

IV. Answer any THREE questions. **(3×4=12)**

19. Show that the curves $r^n = a^n \cos n\theta$ and $r^n = b^n \sin n\theta$ intersect orthogonally.

20. With usual notations prove that $p = r \sin \varphi$ and $\frac{1}{p^2}=\frac{1}{r^2}+\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$

21. Find the radius of curvature of the cycloid $x=a(\theta+\sin\theta), y=a(1+\cos\theta)$

22. Find the circle of curvature for the curve $xy = c^2$ at (c,c).

23. Find all the asymptotes to the curve $x^3+3x^2y-4y^3-x+y+3=0$

V. Answer any THREE questions. **(3×4=12)**

24. Find the reduction formula for $\int \sin^n x\, dx, n>0$.

25. Evaluate $\int_0^{\pi} x \sin^4 x \cos^2 x \, dx$.

26. Find the area of the cardioid $r = a(1+\cos\theta)$

27. Find the length of the arc of the parabola $y^2 = ax$ Cut off by the latus rectum.

28. Find the volume of the solid generated by revolving $r^2 = a^2 \cos 2\theta$ about the line $\theta = \pi/2$