599110

DCMT201

Reg. No. | | | | | | | | |

II Semester B.Sc. Degree Examination, September - 2023

MATHEMATICS - (CORE)

Algebra - II and Calculus - II

Paper : II

(NEP Scheme)

Time : 2½ Hours **Maximum Marks : 60**

Instructions to Candidates:

Answer **all** the questions.

I. Answer any **Six** questions. **(6×2=12)**

1. The binary operation * on the set of positive rational number Q^+ is defined by $a*b=\frac{ab}{2}$ $\forall a,b\in Q^+$, find the identity element of the set Q^+ and the inverse of 3.
2. Find all the left cosets of the subgroup H = {0,2,4} of the group Z_6 w.r.t $\oplus_6$.
3. Prove that every subgroup of an abelian group is a normal subgroup.
4. Define homomorphism of groups.
5. For the curve $r=a(1-\cos\theta)$, find the angle between the radius vector and the tangent.
6. Find the radius of curvature at any point (p,r) on the curve $r^3=a^2p$.
7. Evaluate $\int_0^{\pi/2}\sin^3 x\cos^2 x\,dx$.
8. Write the formula for volume of revolution obtained by revolving y = f(x) between x = a and x = b about the x-axis.

II. Answer any **Three** questions. **(3×4=12)**

9. Prove that $(Z_7,\oplus_7)$ where $Z_7=\{1,2,3,4,5,6\}$ is an abelian group.
10. Prove that a non - empty subset H of a group G is a subgroup of G iff $ab^{-1}\in H\ \forall a,b\in H$.
11. If 'a' is a generator of a cyclic group G, then prove that O(a) = O(G).
12. If G is a cyclic group of order d and G = <a>, then prove that $a^k(k<d)$ is also a generator of G if and only if (k,d) = 1.
13. If H is a subgroup of G, then prove that there exists a one - to -one correspondance between any two right cosets of H in G.

[P.T.O.

III. Answer any **Three** questions. **(3×4=12)**

14. Prove that a subgroup H of a group G is normal iff $ghg^{-1} \in H \ \forall g \in G$ and $h \in H$.
15. If $f: G \to G'$ is defined by $f(x) = -x \ \forall x \in G$, where G is an additive group of integers, then prove that f is isomorphism and find kerf.
16. Prove that product of two normal subgroup of a group is a subgroup of a group.
17. If $f: G \to G'$ is a homomorphism then prove that
    i. $f(e) = e'$ where e and e' are identity elements in G and G' respectively and
    ii. $f(a^{-1}) = [f(a)]^{-1} \forall a \in G$.
18. State and prove Cayley's theorem.

IV. Answer any **Three** questions. **(3×4=12)**

19. Prove that the curves $r = a\sec^2 \theta/2$ and $r = b\operatorname{cosec}^2 \theta/2$ intersect orthogonally.
20. With usual notation prove that $p = r\sin\phi$ and $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$.
21. With the usual notation prove that the radius of curvature of the curve y = f(x) is $\rho = \frac{[1+y_1^2]^{3/2}}{y_2}$.
22. Find all the asymptotes of the curve $x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0$.
23. Prove that the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $(aX)^{2/3} + (bY)^{2/3} = (a^2 - b^2)^{2/3}$.

V. Answer any **Three** questions. **(3×4=12)**

24. Evaluate $\int_0^{\pi} x\cos^6 x\,dx$.
25. Show that length of one loop of the curve $3ay^2 = x(x-a)^2$ is $\frac{4a}{\sqrt{3}}$.
26. Find the surface area of revolution of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis.
27. Find the volume of the solid generated by revolving of the curve cardiod $r = a(1+\cos\theta)$, about the initial line.
28. Evaluate $\int_0^{\infty} \frac{x^2}{(1+x^2)^3}\,dx$.
