



SE – 166

II Semester B.A./B.Sc. Examination, September 2020  
(F + R) (CBCS) (Semester Scheme)  
(2014-15 and Onwards)  
MATHEMATICS (Paper – II)



Time : 3 Hours

Max. Marks : 70

**Instructions :** Answer all Parts.

PART – A

Answer any five questions :

(5×2=10)

1. a) Define subgroup of a group.
- b) In a group  $(G, *) \forall a, b, c \in G$  prove that  $a * b = a * c \Rightarrow b = c$ .
- c) Find the radius of curvature at any point  $(p, r)$  on the curve  $r^3 = a^2 p$ .
- d) Find the length of subtangent to the curve  $r\theta = a$ .
- e) Find  $\frac{ds}{d\theta}$  if  $r^2 = a^2 \cos 2\theta$ .
- f) Write the formula to find the volume of an arc of the curve  $y = f(x)$  from  $x = a$  and  $x = b$ .
- g) Find the integrating factor of  $\frac{dy}{dx} + y \tan x = \sec x$ .
- h) Solve  $p^2 - 5p + 6 = 0$ , where  $p = \frac{dy}{dx}$ .

PART – B

Answer any one full question :

(1×15=15)

2. a) If  $(G, *)$  be a group and  $a, b \in G$ , then prove that  $(a*b)^{-1} = b^{-1} * a^{-1}$ .
- b) Prove that the set of all square roots of unity is a subgroup of the group of fourth roots of unity under multiplication.
- c) Prove that  $G = \{2, 4, 6, 8\}$  is a group under multiplication modulo 10.

OR

P.T.O.



3. a) If  $G$  be the set of rationals except  $-1$  and  $*$  is be the binary operation on  $G$  defined by  $a*b = a + b + ab$ , then prove that  $(G, *)$  is a group.
- b) Prove that  $G = \{1, 5, 7, 11\}$  is an abelian group under multiplication modulo 12.
- c) If  $A = \{1, 2, 3\}$ ,  $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{pmatrix}$  and  $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$  then find  $f \circ f$ ,  $g \circ f$  and  $(g \circ f)^{-1}$ .

## PART – C

Answer **any two full** questions :

(2×15=30)

4. a) With usual notations prove that  $\tan \phi = r \frac{d\theta}{dr}$ , for the polar curve  $r = f(\theta)$ .
- b) Prove that the curves  $r^n = a^n \cos n\theta$  and  $r^n = b^n \sin n\theta$  cut orthogonally.
- c) Show that the radius of curvature at any point on the cardioid  $r = a(1 - \cos \theta)$  is  $\frac{2}{3}\sqrt{2ar}$ .

OR

5. a) With usual notations, prove that the radius of curvature of the curve  $y = f(x)$  is  $\rho = \frac{(1 + y_1^2)^{3/2}}{y_2}$ .
- b) Find the evolute of the ellipse  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .
- c) Obtain the pedal equation of the curve  $r^2 = a^2 \cos 2\theta$ .

6. a) Find all the asymptotes of the curve  $x^3 + x^2y - xy^2 - y^3 - 3x - y - 1 = 0$ .
- b) Find the surface area generated by revolving about the x-axis and the loop of the curve is  $3ay^2 = x(x - a)^2$ .
- c) Find the positions and nature of the double point of the curve,  $x^3 + x^2 + y^2 - x - 4y + 3 = 0$ .

OR

7. a) Find the length of an arc of the cycloid  $x = a(\theta - \sin \theta)$  and  $y = a(1 - \cos \theta)$ .
- b) Find the envelope of the family of lines  $y = mx + \frac{a}{m}$  where  $m$  is a parameter.
- c) Find the volume of the solid generated by revolving the curve astroid  $x^{2/3} + y^{2/3} = a^{2/3}$  about the x-axis.



PART – D

Answer **any one full** question :

(1×15=15)

8. a) Solve  $\frac{dy}{dx} + \frac{1}{x} \sin 2y = x^3 \cos^2 y$ .

b) Verify for exactness and solve  $(4x + 3y + 1)dx + (3x + 2y + 1)dy = 0$ .

c) Solve  $y + px = p^2 x^4$ .

OR

9. a) Solve  $\frac{dy}{dx} - \frac{2}{x}y = (x + x^2)$ .

b) Find the general and singular solution of  $p^2(x^2 - a^2) - 2pxy + y^2 + a^2 = 0$ .

c) Prove that the family  $y^2 = 4a(x + a)$  is self orthogonal.

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