Reg. No.

III Semester B.Sc. Degree Examination, April - 2023

STATISTICS - III

Calculus and Probability Distribution

Paper : III

(NEP Scheme Freshers)

Time : 2½ Hours **Maximum Marks : 60**

Instructions to Candidates:

1. Answer any eight questions from Section A and three questions from Section -B.
2. Scientific calculators are allowed.

SECTION - A

I. Answer any **Eight** questions from the following. **(8×3=24)**

a. Define Jacobian if u = x - y and v = y obtain the Jacobian of x and y w.r.t. u and v.

b. Examine the convergence of the series $p + pq + pq^2 + pq^3 + ...$ where $0 < p < 1$ and $q = 1 - p$.

c. Define joint probability distribution function and mention its properties.

d. Define marginal and conditional probability density functions.

e. If X and Y are independent random variables, show that $M_{X+Y}(t) = M_X(t).M_Y(t)$.

f. State the central limit theorem and mention its applications.

g. State the conditions for the convergence of hyper - geometric distribution to binomial distribution.

h. Define a

   i. Cauchy variate

   ii. Standard Cauchy variate.

i. Define Weibull - Distribution and mention its applications.

j. What is standard error? Write standard error of sample mean.

[P.T.O.

SECTION - B

II. Answer any Three questions from the following. **(3×12=36)**

2. a. State the conditions for continuity and differentiability of a real function.

   b. If $E = \sum_{i=1}^{n}\left(y_i - a - bx_i - cx_i^2\right)^2$ then find partial derivatives of E w.r.t. 'a', 'b' and 'c'.

   c. Integrate $h(x) = xe^{-x}$ over the domain $\{x / 0 < x < \infty\}$ by parts and compare it with the value obtained using gamma function. **(2+6+4)**

3. a. The joint p.d.f of random variables x and y is given by

   $$f(x,y) = x+y \qquad 0<x<1;\ 0<y<1$$
   $$= 0 \qquad \text{otherwise.}$$

   Find marginal distributions of X and Y.

   b. State and prove addition theorem of expectation.

   c. State and prove Chebyshev's inequality. **(3+4+5)**

4. a. Obtain mean and variance of negative binomial distribution.

   b. Define Hypergeometric distribution.

   c. Define Bivariate normal distribution and obtain marginal density function of X. **(5+2+5)**

5. a. Given a random sample $X = (x_1, x_2, ...x_n)$ from $N(\mu, \sigma^2)$ distribution then obtain the sampling distribution of sample mean.

   b. Derive the moment generating function of Chi-square distribution and hence obtain its mean and variance.

   c. State additive property of Chi-square distribution. **(4+6+2)**

6. a. Derive odd ordered central moments of t-distribution with n degrees of freedom.

   b. Obtain mode of F - distribution.

   c. Explain a method of drawing a random sample from exponential distribution with mean θ. **(4+4+4)**