



DCMT301

Reg. No.

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III Semester B.Sc. Degree Examination, April - 2023

MATHEMATICS

Ordinary Differential Equations and Real Analysis - I

Paper : 03

(NEP Scheme Core)



Time : 2½ Hours

Maximum Marks : 60

Instructions to Candidates:*Answer all the questions.***I. Answer any Six questions:**

(6×2=12)

1. Show that the equation $(\sin x \cos y + e^{2x})dx + (\cos x \sin y + \tan y)dy = 0$ is exact.
2. Find the general solution of $(a^2 - x^2)p^2 + 2xyp - y^2 + a^2 = 0$ where $p = \frac{dy}{dx}$.
3. Find the particular integral of $\frac{d^2y}{dx^2} + y = \sin 3x$.
4. Verify the condition for integrability for $(yz + xyz)dx + (zx + xyz)dy + (xy + xyz)dz = 0$.
5. Test the nature of the sequence $\{1 + \cos n\pi\}$.
6. Define a convergent sequence with an example.
7. Test the convergence of the series $1^3 + 2^3 + 3^3 + \dots + n^3$.
8. State the Rabee's test for series of positive terms.

II. Answer any Three questions.

(3×4=12)

9. Verify for exactness and solve $(x^2 - ay)dx + (y^2 - ax)dy = 0$.
10. Solve $xp^2 + (y - x)p - y = 0$.
11. Solve $y = 2px - yp^2$.
12. Find the general and singular solution of $(y - px)(p - 1) = p$.
13. Show that the family of curves $y^2 = 4a(x + a)$ is self orthogonal.

[P.T.O.]



III. Answer any Three questions.

(3×4=12)

14. Solve $y'' - 4y' + 13y = \sin 2x$.

15. Solve $x^2 y'' - xy' + 2y = x \log x$.

16. Solve $\frac{d^2 y}{dx^2} + y = \sec x \tan x$ by the method of variation of parameters.

17. Solve $\frac{dx}{dt} + 2y = -\sin t$, $\frac{dy}{dt} - 2x = \cos t$.

18. Verify the condition for integrability and solve $2yzdx + zxdy - xy(1+z)dz = 0$

IV. Answer any Three questions.

(3×4=12)

19. Discuss the convergence of the sequences whose n^{th} term is

i) $\sqrt{n^2 + 1} - n$

ii) $\frac{1^2 + 2^2 + 3^2 + \dots + n^2}{n^3}$

20. If $\lim_{n \rightarrow \infty} a_n = a$ and $\lim_{n \rightarrow \infty} b_n = b$ then show that $\lim_{n \rightarrow \infty} (a_n \cdot b_n) = a \cdot b$,21. Prove that the sequence $\left\{ \frac{2n-7}{3n+2} \right\}$.

i) is monotonically increasing

ii) is bounded

22. Discuss the nature of the sequence $\left\{ x^n \right\}$ where $x > 0$.23. Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}$ is convergent.

V. Answer any Three questions.

(3×4=12)

24. Discuss the convergence of the series $\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots$.25. Test the convergence of $\sum_{n=1}^{\infty} \frac{1.3.5 \dots (2n-1)}{2.4.6 \dots 2n} \cdot x^n$.

26. State and prove Cauchy's Root test for the series of positive terms.

27. Test the convergence of the series $1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + \dots (p > 0)$.28. Sum to infinity the series $1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \dots$.