11323

Reg. No. | | | | | | | | |

III Semester B.Sc. Degree Examination, March/April - 2021

MATHEMATICS

(CBCS Semester)

Paper : III

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

PART - A

Answer any **five** questions. **(5×2=10)**

1. a) Write the orders of the elements of the group $\{1, w, w^2\}$ with respect to multiplication.

   b) Define right coset and left coset of a group.

   c) Discuss the convergence of the sequence $\frac{\log n}{n}$.

   d) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

   e) State Roabe's test for convergence of series.

   f) Prove that every differentiable function is continueous.

   g) Find the value of 'c', using Rolle's theorem for the function $f(x) = x^2 - 6x + 8$ in [2,4].

   h) Evaluate $\lim_{x \to \infty} \frac{x - \sin x}{x^3}$

PART - B

Answer any **One** full question. **(1×15=15)**

2. a) If 'a' is an element of order 'n' of a group G, 'e' is the identify, then for some positive integer m, $a^m = e$ if and only if 'n' is the devisor of m.

   b) Find all the generators of the cyclic group of order 8.

   c) State and prove Lagrange's theorem for finite groups.

(OR)

[P.T.O.

3. a) Find the order of each element of the group $G=\{0,1,2,3,4,5\}$ under $\oplus_6$.

b) Prove that every subgroup of a cyclic group is cyclic.

c) Find all the distinct right cosets of the subgroup $H=\{0,4,8\}\ in\left(Z_{12},\oplus_{12}\right)$.

PART - C

Answer **two** full questions. **(2×15=30)**

4. a) If $\lim\limits_{n\to\infty} a_n = a$, $\lim\limits_{n\to\infty} b_n = b$. Prove that $\lim\limits_{n\to\infty} a_n b_n = ab$.

b) Discuss the convergence of the sequences

i. $\left\{\dfrac{(-1)^{n-1}}{n}\right\}$

ii. $\left\{\left(\dfrac{n+1}{n}\right)\dfrac{3n^2}{n+1}\right\}$

c) Find the limit of the sequence 0.4,0.44,0.444,.....

(OR)

5. a) Prove that every convergent sequence is bounded.

b) Prove that the sequence $\left\{\dfrac{3n+4}{2n+1}\right\}$ is monotonically decreasing and converges to $\dfrac{3}{2}$.

c) Prove that a monotonically increasing sequence which is bounded above is convergent.

6. a) State and prove Cauchy's root test for the convergence of series of positive terms.

b) Discuss the convergence of the series $\dfrac{1}{3}+\dfrac{2^3}{3^2}+\dfrac{3^3}{3^3}+\dfrac{4^3}{3^4}+.....$

c) Find the sum of infinity of the series

$$1+\frac{4}{6}+\frac{4.5}{6.9}+\frac{4.5.6}{6.9.12}+.....$$

(OR)

7. a) Discuss the convergence of the series

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots..$$

b) Test the absolute, conditional convergence of the series

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\ldots..$$

c) Find the sum to infinity of the series

$$1+\frac{1+5}{2!}+\frac{1+5+5^2}{3!}+\ldots..$$

PART - D

Answer **one** full question. **(1×15=15)**

8. a) Discuss the differentiability of the function at x = 0 if $f(x)=\begin{cases}1+2x for x\le 0\\ 1-3x for x>0\end{cases}$

b) State and prove Lagrange's Mean value Theorem.

c) Evaluate

i. $\lim\limits_{x\to 0}\frac{\log(\tan x)}{\log(x)}$

ii. $\lim\limits_{x\to\frac{\pi}{2}}(\sec x-\tan x)$

(OR)

9. a) Discuss the continuity of the function f(x) at x = 2 if $f(x)=\begin{cases}1+x for x\le 2\\ 5-x for x>2\end{cases}$

b) Find the Maclaurins series expansion of *log(secx)* upto the term containing x^4.

c) Evaluate

i. $\lim\limits_{x\to 0}\frac{a^x-b^x}{x}$

ii. $\lim\limits_{x\to 0}(1+\sin x)^{\cot x}$