12327

Reg. No.

III Semester B.Sc. Degree Examination, April - 2022

STATISTICS - III

Statistical Inference - I

(CBCS Scheme Freshers 2021-2022 and onwards)

Paper - III

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1. Answe any ten sub-divisions from Section - A, any **five** questions from Section - B.
2. Scientific calculators are permitted.

SECTION - A

Answer any **ten** sub-divisions from the following questions. **(10×2=20)**

1. a) Define parameter and statistic.

b) Define scale family of pdf's with an example.

c) Show that the proportion of success in B(n,P) (n is known) is an unbiased estimator of P.

d) Define sufficiency.

e) State invariance property of consistent estimator.

f) Explain the terms :

i. Minimum Variance Unbiased Estimator (MVUE).

ii. Minimum Variance Bound Estimator. (MVBE).

g) State the properties of moment estimators.

h) Explain pivotal quantity method of obtaining confidence interrals.

i) Write 100 $(1-\alpha)$% confidence interval (CI) for Binomial proportion P.

j) Write 100 $(1-\alpha)$% confidence interval (C.I) for variance for small samples.

k) What is meant by simulation?

l) Mention the disadvantages of simulation.

SECTION - B

Answer any **five** questions from the following. **(5×10=50)**

2. a) Derivethe moment generating function (mgf) of Chi-square distribution and hence obtain its mean and variance.

b) Obtain the mode of Chi-square distribution. **(6+4)**

[P.T.O.

3. a) Show that in t - distribution odd order moment vanishes.

   b) Obtain the distribution of reciprocal property of F -variate. (6+4)

4. a) If $X = \{X_1, X_2, \ldots, X_n)$ is a random sample for $N(O, \sigma^2)$ distribution. Show that $T = \frac{1}{n}\sum_{i=1}^{n} x_i^2$ is an unbiased estimator of σ^2.

   b) Show that sample variance $s^2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ is a consistent estimator of population variance σ^2 in $N(\mu, \sigma^2)$ distribution. (5+5)

5. a) Let $X = \{X_1, X_2, \ldots, X_n)$ be a random sample from the probability distribution $f(x,\theta) = \theta . x^{\theta-1};\ 0<x<1,\ \theta > 0$. Show that $\prod_{i=1}^{n} x_i$ is a sufficient estimator of θ.

   b) Given a random sample X_1, X_2, X_3 from a distribution with mean μ and variance σ^2. Obtain the relative efficiency of $T_2 = \frac{5X_1 - 2X_2 + X_3}{4}$ w.r.t. $T_1 = \frac{X_1 + X_2 + X_3}{3}$. (5+5)

6. a) Show that sample mean is a sufficient estimator for 'θ' when the samples drawn from Bernoulli distribution with probability function $P(x) = \theta^x (1-\theta)^{1-x}$, x = 0,1. $0 < \theta < 1$.

   b) A random sample $x_1, x_2, \ldots, x_n$ is drawn from exponential distribution with mean θ. Obtain Minimum Variance Bound (MVB) estimator of θ. (5+5)

7. a) Obtain maximum likelihood estimator of the parameter P in a Binomial Distribution.

   b) Explain the method of moments in the estimation of the parameters and obtain the moment estimator of the parameters 'a' and 'b' in U(a,b) distribution. (4+6)

8. a) Explain the terms :

      i. Confidence co-efficient.

      ii. Shortest confidence interval.

   b) Derive 1000 $(1-\alpha)\%$ confidence interval for the difference of population means $(\mu_1 - \mu_2)$ when

      i. Population variances are known.

      ii. Population variances are unknown. (4+6)

9. a) Describe the method of generating a random sample from Binomial distribution.

   b) Explain the method of generating Random sample from $N(\mu, \sigma^2)$ distribution.(5+5)

11326

Reg. No. | | | | | | | | |

III Semester B.Sc. Degree Examination, April - 2022

STATISTICS - III

Statistical Inference - I

(CBCS Scheme Repeaters 2018 2019-2020 and onwards)

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1. Answer any ten sub-divisions from Section - A, any five questions from Section - B.
2. Answers for section A should be written only in the first four pages of the answer book.

SECTION - A

Answer any **ten** sub-divisions from the following questions. **(10×2=20)**

1. a) Define sampling distribution and standard error.

 b) Show that $MSE(T) = V(T) + (Bias)^2$.

 c) State the desirable quality of a good estimator.

 d) Define unbiased estimator.

 e) State Cramer - Rao inequality.

 f) Define efficient estimator and most efficient estimator.

 g) State any two properties of maximum likelihood estimator (MLE).

 h) What is confidence interval and shortest confidence interval?

 i) Define pivotal quantity? For what purpose it is used?

 j) Write $(1-\alpha)$ 100% confidence interval for the population mean μ of Normal distribution $N(\mu, \sigma^2)$.

 k) What is simulation?

 l) State the advantages and disadvantages of simulation.

[P.T.O.

SECTION - B

Answer any **five** questions from the following. **(5×10=50)**

2. a) Derive the moment generating function of Chi - square distribution.

   b) Establish the additive property of independent Chi-square variates. **(5+5)**

3. a) Show that for t-distribution odd ordered moments are zero.

   b) Obtain the distribution of reciprocal of F - variate. **(5+5)**

4. a) Show that sample mean and sample variance is the unbiased estimator of the population mean and population variance respectively if $X_1, X_2 ...... X_n$ be a random sample from Poisson distribution (λ) P(λ).

   b) If $\overline{X}$ is a mean of sample drawn from Binomial (n,P), then show that $\frac{\overline{X}}{n}$ is a consistent estimator of P. **(5+5)**

5. a) State and prove sufficient condition for consistency.

   b) Define relative efficiency. Let X_1, X_2, X_3 is a random sample from Normal population with mean μ and variance σ^2. Show that the estimators $T_1 = \frac{X_1 + X_2 + X_3}{3}$ and $T_1 = \frac{X_1 + 2X_2 + 3X_3}{6}$ are both unbiased, and compare their efficiency. **(4+6)**

6. a) Define the terms
      i. Minimum variance unbiased estimator (MVUE).
      ii. Minimum variance Bound (MVB).
      iii. Uniformally minimum variance unbiased estimator (UMVUE).

   b) Let $X_1, X_2, ... X_n$ be a random sample of size n from Bernoulli distribution, $f(x,\theta) = \theta^x (1-\theta)^{1-x}$, x = 0,1. Find the sufficient estimator for θ. **(6+4)**

7. a) What is maximum likelihood estimator (MLE)? Find MLE of parameter λ of Poisson distribution.

   b) Find the moment estimators for the parameters α and β of uniform distribution. **(5+5)**

8. a) Obtain $(1-\alpha)$ 100% confidence interval for the population proportion.

   b) Construct confidence interval for population variance of Normal $N(\mu, \sigma^2)$ for small samples. **(5+5)**

9. a) Explain Monte - Carlo method of simulation.

   b) Explain the method of generating random observation from uniform distribution. **(5+5)**