12325

Reg. No.

III Semester B.Sc. Degree Examination, April - 2022

MATHEMATICS

(CBCS Semester Scheme)

Paper : III

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

I. Answer any **Five** questions. **(5×2=10)**

1. Show that 0(-i) = 4 where '-i' belongs to multiplicative group of fourth roots of unity.
2. State any two consequences of Lagrange's theorem on groups.
3. Find the infimum and supremum of $\left\{\frac{n}{n+1}\right\}, n \in N$.
4. Show that the sequence $\{(-1)^n n\}$ is neither bounded above nor below.
5. Show that the series $\sum_{n=1}^{\infty} 2\left(\text{Cos}\frac{\pi}{3}\right)^n$ is convergent and converges to 2.
6. Show that $\sum_{n=1}^{\infty}\left(\frac{3}{4}\right)^{n-1}$ converges to 4.
7. If $L[f(t)] = F(s)$ then show that $L[e^{at} f(t)] = F(s-a)$.
8. Find the inverse Laplace transform of $\frac{s}{4s^2+5}$.

II. Answer any **Three** questions. **(3×5=15)**

9. In a group G, prove that $0(a) = 0(a^{-1})$ where $a \in G$.
10. Prove that every group of order less than or equal to 5 is Abelian.

[P.T.O.

11. Show that every cyclic group is Abelian.

12. Find the order of each element of a group G under addition modulo 5, where $G = \{0,1,2,3,4\}$.

13. if $G = \{1,-1,i,-i\}$ is a group and $H = \{-1,1\}$ be a subgroup of G, list all right and left cosets of H in G. with respect to multiplication.

III. Answer any **three** questions. (3×5=15)

14. If $\{a_n\}$ and $\{b_n\}$ are sequences of real numbers with $\lim_{n\to\infty}\{a_n\} = l$ and $\lim_{n\to\infty}\{b_n\} = m$ then show that $\lim_{n\to\infty}\{a_n + b_n\} = l + m$.

15. Show that the sequence $\left\{\frac{3n+5}{4n+5}\right\}$ is monotonically increasing, bounded and converges to $\frac{3}{4}$.

16. Discuss the convergence of

a. $a_n = \sqrt{n+1} - \sqrt{n}$.

b. $a_n = \dfrac{\log(n+1) - \log n}{\sin\left(\frac{1}{n}\right)}$.

17. Prove that "Every convergent sequence is bounded".

18. Discuss the convergence of the series $\sum\left(\sqrt{n^2+1} - \sqrt{n^2-1}\right)$.

IV. Answer any **two** questions. (2×5=10)

19. State and prove D'Alembert's Ratio test for a series of positive terms.

20. Test the convergence of $\sum_{n=1}^{\infty}\left[\frac{2^n + 3^n}{6^n}\right]$.

21. Find Sum to infinity of the series

$$1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \ldots..$$

V. Answer any **two** questions. (2×5=10)

22. Find the Laplace transform of

a. $Cosh^2 3t$.

b. 5^t.

23. Show that $L[t^n]=\frac{n!}{s^{n+1}}$ where 'n' is positive integer.

24. Verify the convolution theorem on Laplace transforms for the pair of functions $f(t)=t$ and $g(t)=\cos t$.

VI. Answer any **two** questions. (2×5=10)

25. A child building a tower with blocks uses 15 for the bottom row. Each row has two fever blocks than the previous row. If there are eight rows in the tower, find the total number of blocks in the tower.

26. A voltage $E.e^{-at}$ is applied at t=0 to a circuit of inductance L and resistance R. Show by Laplace transform method that current at any time t is $\frac{E}{R-aL}\left[e^{-at}-e^{-\left(\frac{R}{L}\right)t}\right]$.

27. Using Laplace transform, solve the differential equation, $\frac{dy}{dx}+5y-e^{-5x}=0$ given y(0) =2.

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Reg. No. | | | | | | | | |

III Semester B.Sc. Degree Examination, April - 2022

MATHEMATICS

(CBCS Semester Scheme 2018 Batch and onwards)

Paper : III

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

PART - A

Answer any **five** questions. **(5×2=10)**

1. a) Define right coset of a subgroup of a group.

   b) Find all the left cosets of the subgroup $H = \{0,2,4\}$ of the group $(Z_6, \oplus_6)$.

   c) State Raabe's test.

   d) Show that $\left\{\frac{1}{n}\right\}$ is monotonically decreasing sequence.

   e) Test the convergence of the series

   $$\sqrt{\frac{1}{4}}+\sqrt{\frac{2}{6}}+\sqrt{\frac{3}{8}}+.....$$

   f) State Cauchy's mean value theorem.

   g) Verify Rolle's theorem for the function $f(x) = 8x - x^2$ in [2,6].

   h) Evaluate $\underset{x\to 0}{lt}\left(\frac{1-\cos x}{x^2}\right)$ by using L - Hospitals' rule.

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PART - B

Answer **one** full question. **(1×15=15)**

2. a) In a group G, prove that $O(a) = O(a^{-1}), \forall a \in G$.

b) Find all the generators of the cyclic group of order 8.

c) Find all the distinct right cosets of the subgroup $H = \{0,4,8\}$, in $(Z_{12}, \oplus_{12})$.

(OR)

3. a) Prove that any two right (left) cosets of subgroup H of a group G are either identical or disjoint.

b) Prove that every group of order Four is abelian.

c) If an element 'a' of a group G is of order 'n' and e is the identity in G, then prove that for some positive integer m, $a^m = e$ if and only if n divids m.

PART - C

Answer **two** full questions. **(2×15=30)**

4. a) If $\lim_{n\to\infty} a_n = a$ and $\lim_{n\to\infty} b_n = b$, prove that $\lim_{n\to\infty}(a_n + b_n) = a + b$.

b) Discuss the nature of the sequence $\left\{\left(1+\frac{1}{n}\right)^n\right\}$.

c) Test the convergence of the sequence

i. $\left\{\frac{3n-4}{4n+3}\right\}$ ii. $\left\{\frac{(-1)^{n-1}}{n}\right\}$.

(OR)

5. a) Prove that every convergent sequence is bounded.

b) Examine the convergence of the sequence

i. $\left\{\left(\frac{2n^2+3n+5}{n+3}\right) Sin\left(\frac{\pi}{n}\right)\right\}$ ii. $\left\{\left(1+\frac{a}{n}\right)^{\frac{n}{b}}\right\}$

c) Prove that a sequence which is monotonically increasing and bounded above is convergent.

6. a) State and prove D'Alembert's ratio test for series of positive terms.

b) Test the convergence of the series.

$$\frac{1}{1.2.3}+\frac{3}{2.3.4}+\frac{5}{3.4.5}+.....$$

c) Find the sum to infinity of the series $1+2\left(\frac{1}{9}\right)+\frac{2.5}{1.2}\left(\frac{1}{81}\right)+\frac{2.5.8}{1.2.3}\left(\frac{1}{729}\right)+.....$

(OR)

7. a) State and prove Cauchy's root test for a series of positive terms.

b) Discuss convergence of the series

$$\sum\frac{1.2.3.....n}{3.5.7.9.....(2n+1)}.$$

c) Sum to infinity the series

$$1+\frac{1+2}{\lfloor 2}+\frac{1+2+3}{\lfloor 3}+\frac{1+2+3+4}{\lfloor 4}+.....$$

PART - D

Answer **one** full questions. **(1×15=15)**

8. a) Prove that every continuous function over a closed interval is bounded.

b) Evaluate $\underset{x\to 0}{lt}(1+\sin x)^{\cot x}$ using L - Hospital's rule.

c) Expand the function $\log_e^{(1+x)}$ up to the term containing x^4 by Maclaurin's expanison.

(OR)

9. a) State and prove Lagranges mean value theorem.

b) Examine the differentiability of the function $f(x)=\begin{cases}x^2 & if\ x\leq 3\\ 6x-9 & if\ x>3\end{cases}$ at x = 3.

c) Evaluate $\lim_{x\to 0}\left(\frac{1}{x}\right)^{\tan x}$ by using L - Hospital's rule.