12423

Reg. No.

IV Semester B.Sc. Degree Examination, September/October - 2022

MATHEMATICS

(CBCS Scheme Semester 2020 Regular)

Paper : IV

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

I. Answer any **Five** questions. **(5×2=10)**

1. Define centre of a group.

2. Show that $f:(R,+)\to(R,+)$ defined by $f(x)=2^x$ is not a homomorphism.

3. Write the half range fourier cosine expansion for f(x) over $(0,l)$.

4. State Lagrange's mean value theorem.

5. Evaluate $\lim_{x\to 0}\left(\frac{x-\sin x}{x^2}\right)$.

6. Write the Maclaurin's expansion for *f(x,y)*.

7. Find the particular integral of the equation $\left(D^3+D^2-D-1\right)y=e^x$.

8. Find a part of complimentary function of the equation $\frac{d^2y}{dx^2}-\frac{1}{x-1}\left(\frac{dy}{dx}\right)-\frac{y}{x-1}=x-1$.

II. Answer any **Two** questions. **(2×5=10)**

1. Prove that the product of any two normal subgroups of a group is a subgroup of the group.

[P.T.O.

2. If $f : G \to G'$ is a homomorphism of groups with kernel k then prove that k is a normal subgroup of G.

3. Let S = {1,2,3,4} and $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$, $g = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ show that $fog \neq gof$.

III. Answer any **Two** questions. (2×5=10)

1. Expand the function $f(x) = x^2$ as a fourier series in the interval $-\pi \le x \le \pi$ and hence deduce that $\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$

2. Find the fourier series of the periodic function defined by $f(x) = 2x - x^2$ in the interval $0<x<3$.

3. Obtain the half range fourier sine series of $f(x) = (x-1)^2$ in the interval $(0,1)$.

IV. Answer any **Three** questions. (3×5=15)

1. Discuss the continuity of the function $f(x) = \begin{cases} 1+x & for\ x \le 2 \\ 5-x & for\ x \ge 2 \end{cases}$ at x = 2.

2. State and prove Rolle's theorem.

3. Expand $\log(1+x)$ by Maclaurin's expansion.

4. Find the extreme value of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.

5. Find the extremum of xy^2z^3 subjected to the condition $x+y+z=6$.

V. Answer any **Three** questions. (3×5=15)

1. Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 4y = e^x \sin x$.

2. Solve $x^2y'' - xy' + 2y = x\log x$.

3. Solve $\sin^2 xy''+\sin x\cos xy'+4y=0$ by changing the independent variable.

4. Solve $y''+a^2y=\sec ax$ by the method of variation of parameters.

5. Verify that equation $(1-x^2)y''-3xy'-y=0$ is exact and solve it.

VI. Answer any **Two** questions. **(2×5=10)**

1. An alternating current after passing through a full wave rectifier has the form $I=I_0|\sin t|$, $0<t<2\pi$, where I_0 is the maximum current. Express I as a fourier series.

2. An alternating current after through a rectifier has the form $I=\begin{cases} I_0\sin\theta & for\, 0<\theta\le\pi \\ 0 & for\ \pi<\theta\le 2\pi\end{cases}$ where I_0 is the maximum current. Express I as fourier series in $(0,2\pi)$.

3. If F(t) is the periodic function and is defined in one period as $F(t)=\begin{cases} 1 & for\, 0<t\le\pi \\ 0 & for\ \pi<t\le 2\pi\end{cases}$ Find the solution of the differential equation $y''-y=F(t)$.

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IV Semester B.A./B.Sc. Degree Examination, September/October - 2022

MATHEMATICS

Mathematics - IV

(CBCS Scheme 2018-19 Repeaters 2015-16 Onwards)

Paper : IV

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1. Answer all the questions.
2. Non - programmable scientific calculators are allowed.

PART - A

Answer any **Five** questions. **(5×2=10)**

1. a. Define normal subgroup of a group.
   b. Define homomorphism of groups.
   c. Find a_0 in the fourier series of $f(x)=e^x$ in $(-\pi,\pi)$.
   d. Write Taylor's series expansion for the function f(x,y) about the point (a,b).
   e. Find $L[\cos^2 t]$.
   f. Find $L^{-1}\left[\frac{2s}{s^2+16}\right]$.
   g. Solve : $(D^2+4D+2)y=0$.
   h. Find particular integral of $(D^2+4D+4)y=e^{2x}$.

PART - B

Answer **one** full questions. **(1×15=15)**

2. a. Prove that the centre of a group G is a normal sub group of G.
   b. Prove that subgroup H of a group G is normal iff $gHg^{-1}=H, \forall g \in G$.
   c. If $G \to G'$ is a homomorphism, then prove that f(G) is a subgroup of G'.

(OR)

3. a. If H is a subgroup of G and K is a normal subgroup of G then prove that $H \cap K$ is a normal subgroup of G.
   b. State and prove fundamental theorem of homomorphism.
   c. If $f: G \to G'$ is a homomorphism from group G into group G' with kernel k, then prove that f is one - one iff $k=\{e\}$ where e is the identify element of G.

PART - C

[P.T.O.

Answer any **Two** full questions. (2×15=30)

4. a. Obtain the fourier series of $f(x)=x^2$ in $(-\pi,\pi)$.

b. Find the half range cosine series of $f(x)=2x-1$ in (0,2).

c. Expand $x^2y+3y-2$ in powers of (x-1) and (y+2) using Taylor's theorem.

(OR)

5. a. Test for maximum and minimum of the function $f(x,y)=x^2+y^2-3xy$.

b. Find the three numbers x,y,z such that $x+y+z=1$ and $xy+yz+zx$ is maximum.

c. Expand $f(x)=(x-1)^2$ in 0<x<1 in terms of half - range sine series.

6. a. Prove that $L[\text{cosh}mt]=\frac{s}{s^2-m^2}$.

b. Find the value of $L[t.\sin 2t]$.

c. Find $L^{-1}\left[\frac{s}{(s-3)(s^2+4)}\right]$.

(OR)

7. a. Find $L^{-1}\left[\frac{1}{(s^2+a^2)^2}\right]$.

b. Find $L\left[\frac{e^{-at}-e^{-bt}}{t}\right]$.

c. Solve $y''+9y=25e^{4t}$ given y(0) = 3 and $y'(0)=7$ by using laplace transform.

PART - D

Answer **one** full questions. (1×15=15)

8. a. Solve : $\left(D^2+1\right)y=e^{-x}+5e^{2x}$.

b. Solve : $4x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}-y=4x^2$.

c. Solve $x^2y''+xy'-y=2x^2(x>0)$ given that 1/x is a part of the complementary function and y(1) $y(1)=y'(1)=0$.

(OR)

9. a. Solve : $x^2y_2+x^2y_1-y=x^2e^x;x>0$, by the method of variation of parameters.

b. Solve : $\frac{dx}{dt}-7x+y=0;\ \frac{dy}{dt}-2x=5y$.

c. Solve : $\left(D^2-2D+4\right)y=e^x\cos x$.