636636

DCST401

Reg. No.

IV Semester B.Sc. Degree Examination, September - 2023

STATISTICS

Statistical Inference - I

Paper : 4

(NEP 2020 Scheme)

Time : 2½ Hours **Maximum Marks : 60**

Instructions to Candidates:

1. Answer any **Eight** sub-divsions from Section-A and **three** questions from Section-B.
2. Scientific **calculators** are allowed.

SECTION - A

I. Answer any **EIGHT** sub-divisions from the following. **(8×3=24)**

1. a. Define location family of P.d.f's with an example.

   b. Distinguish between an 'estimator' and an 'estimate'. Mention the desirable qualities of an estimator.

   c. Define asymptotic unbiasedness.

   Show that $S^2 = \frac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$ is asymptotically unbiased for σ^2, when the sample is taken from $N(\mu, \sigma^2)$ distribution.

   d. If T is is an estimator of θ, obtain MSE (T) interms of variance and bias of the estimator.

   e. Define 'Likelihood function'. Explain the method of maximum likelihood in point estimation.

   f. State the properties of moment estimators.

   g. State Cramer - Rao inequality and mention its utility.

   h. What are simple and compositive hypotheses? Classify the following hypotheses accordingly?

   i. $H: \mu = \mu_0$, in $N(\mu, \sigma_0^2)$, where σ_0^2 is known distribution.

   ii. $H: \lambda = \lambda_0$, in $P(\lambda)$ distribution.

   iii. $H: \theta \geq \theta_0$ in geometric distribution with parameter θ.

[P.T.O.

i. Explain
   i. Test function.
   ii. Randomized and non - randomized tests.

j. Explain t-test for testing single mean.

SECTION - B

II. Answer any **THREE** questions from the following. **(3×12=36)**

2. a. Derive the sufficient conditions for the consistency of the estimators.
   b. Show that in a normal distribution $N(\mu,\sigma^2)$, the sample mean is more efficient than sample median.
   c. Obtain sufficient statistic of λ in Poisson (λ) distribution. **(3+5+4)**

3. a. Obtain maximum likelihood estimator of θ in $f(x;\theta)=\frac{1}{\theta}e^{-x/\theta}$; $0<x<\infty$.
   b. Obtain moment estimator of λ in P(λ) distribution.
   c. Obtain MVB estimator of μ in normal $N(\mu,\sigma_0^2)$ distribution, where σ_0^2 in known. **(4+3+5)**

4. a. If $C=\{x/x<1\}$ is the critical region of a test based on single observation X for testing $H_0:\theta=2$ against $H_1:\theta=1$, θ being the parameter of exponential distribution with mean $\frac{1}{\theta}$, compute probabilities of two types of errors. Also find power of the test.
   b. State NP lemma. Using it obtain MP test of level 'α' based on a random sample of size 'n' from a Poisson distribution with parameter λ for testing $H_0:\lambda=\lambda_0$ against $H_1:\lambda=\lambda_1$ $(\lambda_1>\lambda_0)$. **(6+6)**

5. a. Explain large sample test procedure for testing the equality of two population proportions.
   b. Explain paired t-test.
   c. Stating the assumptions involved, describe the procedure for testing $H_0:\sigma_1^2=\sigma_2^2$ against $H_1:\sigma_1^2\neq\sigma_2^2$. **(4+3+5)**

6. a. Derive $(1-\alpha)100\%$ confidence intervals for the difference of population means $(\mu_1-\mu_2)$ when population variances are known.
   b. Obtain $(1-\alpha)100\%$ confidence intervals for variance σ^2 of a normal population when
      (i) μ is known.
      (ii) μ is unknown. **(5+7)**