561060

DCMT401

Reg. No. [ ]

IV Semester B.Sc. Degree Examination, September - 2023

MATHEMATICS

Partial Differential Equations and Integral Transforms

Paper : IV

(NEP Scheme)

Time : 2½ Hours **Maximum Marks : 60**

Instructions to Candidates:

Answer **all** the questions.

I. Answer any SIX questions. **(6×2=12)**

1. Form partial differential equation by eliminating arbitrary function from $z = f(x^2 - y^2)$.
2. Solve $pq + p + q = 0$.
3. Find the complimentary function for $\left(D^2 - 4DD^1 + 4D^{12}\right)Z = 0$.
4. Write the formula to find the solution of one - dimensional heat equation.
5. Find $L\{\cos^2 2t\}$.
6. If $L\{f(t)\} = F(S)$ then prove that $L\{f(t)\} = SF(S) - f(o)$.
7. Find the Fourier co-efficient b_n for the function $f(x) = x + x^2$ in $(-\pi, \pi)$.
8. Write Half range Fourier cosine series formula.

II. Answer any **THREE** questions. **(3×4=12)**

9. Form the partial differential equation by eliminating arbitrary function from $z = e^{ax+by} f(ax - by)$.
10. Solve $(mz - ny)p + (nx - lz)q = ly - mx$.
11. Solve $p^2 + q^2 = x + y$.
12. Solve $z^2(p^2 + q^2 + 1) = 1$.
13. Solve the Charpits method $px + qy = pq$.

III. Answer any **THREE** questions. **(3×4=12)**

14. Solve $\left(D^2 - DD^1\right)Z = Sin\ x\ Cos2y$.
15. Solve $(D+1)(D + D^1 - 1)z = \sin(x + 2y)$.

[P.T.O.

16. Reduce $\frac{\partial^2 z}{\partial x^2} = x^2\left(\frac{\partial^2 z}{\partial y^2}\right)$ to a canonical form.

17. A tightly stretched string with fixed end points $x=0$ and $x=l$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its point a velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 3(lx - x^2)$ find y(x,t).

18. Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ subject to the condition

   i) u(0,t) = 0, u(1,t) = 0 for all t.

   ii) u(x,0) = $x^2 - x$; $0 \le x \le 1$.

IV. Answer any **THREE** questions. (3×4=12)

19. Find

   i. $L\{\sin 5t . \cos 2t\}$.

   ii. $L\{e^{2t} \cos^2 t\}$

20. If L{f(t)} = F(S) then prove that $L\left\{\frac{f(t)}{t}\right\} = \int_s^\infty F(s)\,ds$. Hence evaluate $L\left\{\frac{\sin t}{t}\right\}$.

21. Find $L^{-1}\left\{\frac{S+5}{(S-1)(S^2+4)}\right\}$.

22. Verify the convolution theorem for the function f(t) = t ; g(t) = cost.

23. If f(t) = t^2, 0<t<2 and f is periodic function of period 2 then find L{f(t)}.

V. Answer any **THREE** questions. (3×4=12)

24. Obtain the Fourier series for the function f(x) = e^x in $(-\pi, \pi)$.

25. If $f(x) = \left(\frac{\pi - x}{2}\right)^2$ in $0 < x < 2\pi$. Find the Fourier series.

26. Find the Fourier half range sine series of $f(x) = x^2$ in $0 < x < \pi$.

27. Find the Fourier integral expansion of $f(x) = e^{-ax}$, x>0 and $f(-x) = f(x)$, a>0.

28. Find the Fourier cosine transform of $f(x) = \begin{cases} 1, & 0 \le x < a \\ 0, & x \ge a \end{cases}$.