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GS-322

IV Semester B.A./B.Sc. Examination, May/June - 2019

MATHEMATICS

Mathematics - IV

(CBCS) (F+R) (2015-16 & Onwards)

Time : 3 Hours　　　　Max. Marks : 70

Instructions : *Answer **all** Parts.*

PART-A

1. Answer **any five** questions :　　　　**5x2=10**
   - (a) Define normal subgroup of a group.
   - (b) Verify whether $f : (Z, +) \to (Z, +)$ defined by $f(n) = 4n + 2$ is a homomor phism.
   - (c) Obtain half range sine series of $f(x) = 2x - 1$ in the interval $(0, 1)$.
   - (d) Write the necessary conditions for $f(x, y)$ to have extreme values at (a, b).
   - (e) Find $L[\cos^2(2t)]$
   - (f) Find $L^{-1}\left[\dfrac{2S - 1}{S^2 + 16}\right]$
   - (g) Solve $(D^2 + 4D + 2)y = 0$.
   - (h) Verify whether $(1 - x^2)y'' - 3xy' - y = 0$ is exact.

PART-B

Answer **one** full question :　　　　**1x15=15**

2. (a) Prove that a subgroup H of a group G is a normal subgroup of G if and only if every left coset of H is also a right coset of H in G.

   (b) If N is a normal subgroup of a group G and G/N is a collection of all cosets of N in G.

   Prove that G/N is a group under the binary operation $(N_a)(N_b) = N_{ab}, \forall\ N_a, N_b \in G/N$

   (c) If $f : G \to G$ is a homomorphism of a group G into itself and H is a cyclic subgroup of G then prove that $f(H)$ is also a cyclic subgroup of G.

OR

3. (a) Prove that the product of any two normal subgroups of a group is again a normal subgroup.

   (b) Let $G = \left\{\dfrac{a+b\sqrt{2}}{a, b \in Q}\right\}$. Show that $f : (G, +) \to (G, +)$ defined by $f(a+b\sqrt{2}) = a-b\sqrt{2}$ is an isomorphism.

   (c) State and prove Cayley's theorem.

P.T.O.

PART-C

Answer **any two** full questions : **2x15=30**

4. (a) Obtain Fourier series expansion of the function $f(x) = \begin{cases} -x \text{ for } -\pi < x < 0 \\ x \text{ for } 0 < x < \pi \end{cases}$

(b) Obtain Fourier series expansion of the function $f(x) = x - x^2$ in the interval $(-1, 1)$.

(c) Find Taylor's series expansion of $f(x, y) = \dfrac{y^2}{x^3}$ about the point $(1, -1)$ upto 2^{nd} degree terms.

OR

5. (a) Find the maximum and minimum distances of the point (1,2,3) from the sphere $x^2 + y^2 + z^2 = 56$ using Lagrange's method.

(b) Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$

(c) Find the half range Fourier cosine series of $f(x) = \sin x$ in the interval $(0, \pi)$.

6. (a) (i) If $L[f(t)] = F(S)$ then prove that $L[f(at)] = \dfrac{1}{a} F\left(\dfrac{s}{a}\right)$.

(ii) Find $L[e^{-t}(2\sin 2t.\ \cos 5t)]$

(b) Find the Laplace transform of the function

$f(t) = \begin{cases} 1, & 0<t<T \\ -1, & T<t<2T \end{cases}$ given that f(t) is periodic with period 2T.

(c) Find $L^{-1}\left[\dfrac{S+5}{(S-1)(S^2+4)}\right]$

OR

7. (a) If $L[f(t)] = F(S)$ then prove that $L\left[\dfrac{f(t)}{t}\right] = \int_S^\infty F(S)ds$. Hence evaluate $L\left[\dfrac{\sin t}{t}\right]$

(b) Find $L^{-1}\left[\log\left(\dfrac{S^2+1}{S(S+1)}\right)\right]$

(c) State convolution theorem and use it to find $L^{-1}\left[\dfrac{S}{(S^2+a^2)^2}\right]$

PART-D

Answer **one** full question : **1x15=15**

8. (a) Solve $y''+3y'+2y=\cos^2 x$

(b) Solve $x^2y''-xy'+2y=x\log x$

(c) solve $xy''-(2x+1)y'+(x+1)y=x^2e^{2x}$ given that e^x is a part of the complementary function.

OR

9. (a) Solve $(D^2-2D+4)y=e^x\cos x$

(b) Solve $\dfrac{dx}{dt}+7x-y=0;\quad \dfrac{dy}{dt}+2x+5y=0$

(c) Solve $x\dfrac{d^2y}{dx^2}-\dfrac{dy}{dx}-4x^3y=8x^3\sin(x^2)$ using the transformation $z=x^2$.

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