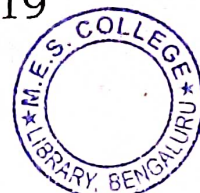


**GS-322**

IV Semester B.A./B.Sc. Examination, May/June - 2019

MATHEMATICS
Mathematics - IV

(CBCS) (F+R) (2015-16 & Onwards)



Time : 3 Hours

Max. Marks : 70

Instructions : Answer all Parts.**PART-A**1. Answer **any five** questions :**5x2=10**

- (a) Define normal subgroup of a group.
- (b) Verify whether $f : (Z, +) \rightarrow (Z, +)$ defined by $f(n) = 4n + 2$ is a homomorphism.
- (c) Obtain half range sine series of $f(x) = 2x - 1$ in the interval $(0, 1)$.
- (d) Write the necessary conditions for $f(x, y)$ to have extreme values at (a, b) .
- (e) Find $L[\cos^2(2t)]$
- (f) Find $L^{-1}\left[\frac{2S - 1}{S^2 + 16}\right]$
- (g) Solve $(D^2 + 4D + 2)y = 0$.
- (h) Verify whether $(1 - x^2)y'' - 3xy' - y = 0$ is exact.

PART-BAnswer **one** full question :**1x15=15**

2. (a) Prove that a subgroup H of a group G is a normal subgroup of G if and only if every left coset of H is also a right coset of H in G .
- (b) If N is a normal subgroup of a group G and G/N is a collection of all cosets of N in G .

Prove that G/N is a group under the binary operation $(N_a)(N_b) =$

$$N_{ab}, \forall N_a, N_b \in G/N$$

- (c) If $f : G \rightarrow G$ is a homomorphism of a group G into itself and H is a cyclic subgroup of G then prove that $f(H)$ is also a cyclic subgroup of G .

OR

3. (a) Prove that the product of any two normal subgroups of a group is again a normal subgroup.

- (b) Let $G = \left\{ \frac{a+b\sqrt{2}}{a, b \in \mathbb{Q}} \right\}$. Show that $f : (G, +) \rightarrow (G, +)$ defined by

$$f(a+b\sqrt{2}) = a-b\sqrt{2} \text{ is an isomorphism.}$$

- (c) State and prove Cayley's theorem.

P.T.O.



PART-C

Answer any two full questions :

2x15=30

4. (a) Obtain Fourier series expansion of the function $f(x) = \begin{cases} -x & \text{for } -\pi < x < 0 \\ x & \text{for } 0 < x < \pi \end{cases}$
- (b) Obtain Fourier series expansion of the function $f(x) = x - x^2$ in the interval $(-1, 1)$.
- (c) Find Taylor's series expansion of $f(x, y) = \frac{y^2}{x^3}$ about the point $(1, -1)$ upto 2nd degree terms.

OR

5. (a) Find the maximum and minimum distances of the point $(1, 2, 3)$ from the sphere $x^2 + y^2 + z^2 = 56$ using Lagrange's method.
- (b) Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$
- (c) Find the half range Fourier cosine series of $f(x) = \sin x$ in the interval $(0, \pi)$.

6. (a) (i) If $L[f(t)] = F(S)$ then prove that $L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$.
- (ii) Find $L[e^{-t}(2\sin 2t \cdot \cos 5t)]$
- (b) Find the Laplace transform of the function
- $$f(t) = \begin{cases} 1, & 0 < t < T \\ -1, & T < t < 2T \end{cases} \text{ given that } f(t) \text{ is periodic with period } 2T.$$
- (c) Find $L^{-1}\left[\frac{S+5}{(S-1)(S^2+4)}\right]$

OR

7. (a) If $L[f(t)] = F(S)$ then prove that $L\left[\frac{f(t)}{t}\right] = \int_S^\infty F(S) ds$. Hence evaluate $L\left[\frac{\sin t}{t}\right]$
- (b) Find $L^{-1}\left[\log\left(\frac{S^2+1}{S(S+1)}\right)\right]$
- (c) State convolution theorem and use it to find $L^{-1}\left[\frac{S}{(S^2+a^2)^2}\right]$

**PART-D**Answer **one** full question :**1×15=15**

8. (a) Solve $y'' + 3y' + 2y = \cos^2 x$
(b) Solve $x^2 y'' - xy' + 2y = x \log x$
(c) solve $xy'' - (2x+1)y' + (x+1)y = x^2 e^{2x}$ given that e^x is a part of the complementary function.

OR

9. (a) Solve $(D^2 - 2D + 4)y = e^x \cos x$
(b) Solve $\frac{dx}{dt} + 7x - y = 0$; $\frac{dy}{dt} + 2x + 5y = 0$
(c) Solve $x \frac{d^2 y}{dx^2} - \frac{dy}{dx} - 4x^3 y = 8x^3 \sin(x^2)$ using the transformation $z = x^2$.

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