SE – 167

IV Semester B.A./B.Sc. Examination, September 2020
(CBCS) (Semester Scheme) (F + R) (2015-16 and Onwards)
MATHEMATICS (Paper – IV)

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** Parts.*

PART – A

Answer **any five** questions. **(5×2=10)**

1. a) Prove that every subgroup of an abelian group is normal.

   b) Verify whether $f : G \to G'$ defined by $f(x) = 2^x$ is homomorphism or not.

   c) Find a_0 in the Fourier series of $f(x) = e^{-ax}$ in $(-\pi, \pi)$.

   d) Show that $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ is maximum at $(-1, -2)$.

   e) Find the Laplace transform of cos5tsint.

   f) Find $L^{-1}\left[\dfrac{s+2}{s^2-2s+5}\right]$.

   g) Solve : $\dfrac{d^4y}{dx^4}-16y=0$

   h) Find the value of 'y' from the simultaneous equations $\dfrac{dx}{dt}+7x-y=0$ and $\dfrac{dy}{dt}+2x+5y=0$.

PART – B

Answer **one full** question. **(1×15=15)**

2. a) Prove that the product of any two normal subgroups of a group is again a normal subgroup.

   b) Let $f : G \to G'$ be an homomorphism of a group G into group G′ with kernel K. Then f is one-one if and only if $K = \{e\}$, where 'e' is the identify in G.

   c) If $f : (z_8, t_8) \to (z_2, t_2)$ is given by $f(x) = r$ where r is the remainder when x is divided by 2. Show that f is homomorphism.

OR

P.T.O.

3. a) Prove that a subgroup H of a group G is normal if and only if every right coset of H in G is a left coset of H in G.
   b) Let G be a group and H be a normal subgroup of G, then prove that G/H is a homomorphic image of G with H as its Kernel.
   c) State and prove fundamental theorem of homomorphism.

PART – C

Answer **any two full** questions. **(2×15=30)**

4. a) Find the Fourier series of $f(x) = 1 - x^2$ in $-1 \leq x \leq 1$.
   b) Obtain the Fourier half range cosine series for the function 'f' defined by $f(x) = \sin x$ in $(0, \pi)$.
   c) Obtain Taylor's expansion of $\tan^{-1}(y/x)$ about the point (1, 1) upto second degree terms.

OR

5. a) Find the extreme values of the function $f(x, y) = 1 + \sin(x^2 + y^2)$.
   b) Show that a rectangular box of maximum volume with prescribed surface area is a cube.
   c) Obtain the Fourier series for e^x in the interval $(-\pi, \pi)$.

6. a) Find the Laplace transform of $e^{2t}(2t^2 - 3t + 4)$.
   b) Find $L\{f(t)\}$ if $f(t) = \begin{cases} t & 0 < t < 4 \\ 5 & t > 4 \end{cases}$.
   c) Find $L^{-1}\left[\dfrac{2s^2 - 6s + 5}{(s-1)(s-2)(s-3)}\right]$.

OR

7. a) Find $L\left[\dfrac{\sin^2 t}{t}\right]$.
   b) By using the convolution theorem prove that $L\left\{\int_0^t f(t)dt\right\} = \dfrac{1}{s}L\{f(t)\}$.
   c) Find $L^{-1}\left[s\log\left(\dfrac{s+4}{s-4}\right)\right]$.

PART – D

Answer **one full** question. (1×15=15)

8. a) Solve : $(2D^2 + 2D + 3)y = x^2 + 2x - 1$.

   b) Solve $x^2y'' + xy' - 9y = 0$ given that x^3 is a part of the complimentary function.

   c) Solve : $y'' + 2y' + 5y = e^{-x} \sin 2x$.

OR

9. a) Solve : $x^2 \frac{d^2y}{dx^2} - 3x\frac{dy}{dx} + 4y = (1+x)^2$.

   b) Solve the simultaneous differential equation $\frac{dx}{dt} = 3x - y$ and $\frac{dy}{dt} = x + y$.

   c) Solve : $x^2y_2 + xy_1 - y = x^2e^x$, $x > 0$ by the method of variation of parameters.