11525

Reg. No. | | | | | | | | |

V Semester B.Sc. Degree Examination, April - 2022

MATHEMATICS

(CBCS Scheme Semester 2022)

Paper : V

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

PART - A

Answer any **five** questions. **(5×2=10)**

1. a) In a Ring $(R,+,\cdot)$ prove that $(-a).(-b) = a.b \ \forall a,b \in R$.

   b) Define subring of a ring and given an example.

   c) Prove that every field is a principal ideal ring.

   d) Find the maximum directional derivative of $\phi = x^3y^2z$ at the point (1,-2,3).

   e) If $\overline{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ then show that $\overline{F}$ is irrotational vector.

   f) Evaluate : $\Delta^4(1-ax)(1-bx)(1-cx)(1-dx)$.

   g) Write the Newton's divided difference interpolation formula.

   h) State the Trapezoidal rule for the integral $\int_a^b f(x)dx$.

PART - B

Answer **two** full questions. **(2×10=20)**

2. a) Prove that the set R = {0,1,2,3,4,5} is a commutative ring with respect to $\oplus_6$ and $\otimes_6$ as the two composition.

   b) Prove that a ring R is without zero divisors if and only if the cancellation laws hold in R.

OR

3. a) Prove that the ring $(Z_n, +_n, X_n)$ is an integral domain if and only if n is a prime number.

   b) Show that the set of all real numbers of the form $a + b\sqrt{2}$ where a and b are integers is a ring with respect to addition and multiplication.

[P.T.O.

4. a) Find all the principal ideals of the ring $R = \{0,1,2,3,4,5,6,7\}$ with respect to $\oplus_8$ and $\otimes_8$.

b) If $f : R \to R'$ be a homomorphism with kernal k, then prove that f is one - one if and only if $k = \{0\}$.

(OR)

5. a) Let $R = R' = C$ be the field of complex numbers, Let $f : R \to R'$ be defined by $f(Z) = \overline{Z}$ where $\overline{Z}$ is the complex conjugate of Z, show that f is an isomorphism.

b) State and prove fundamental theorem of homomorphism of rings.

PART - C

Answer **two** full questions. **(2×10=20)**

6. a) Find the directional derivative of $\phi(x,y,z) = x^2 - 2y^2 + 4z^2$ at the point (1,1,-1) in the direction of $2\hat{i} - \hat{j} + \hat{k}$.

b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$ then prove that $\nabla r^n = nr^{n-1}\vec{r}$.

(OR)

7. a) Find the angle between the surfaces $4x^2y + z^3 = 4$ and $5x^2y - 2yz = 9x$ at the point (1,-1,2).

b) If the vector $\overline{F} = (3x+3y+4z)\hat{i} + (x - ay + 3z)\hat{j} + (3x + 2y - z)\hat{k}$ is solenoidal find 'a'.

8. a) Prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ where $r^2 = x^2 + y^2 + z^2$.

b) Show that the vector $\overline{F} = (6xy + z^3)\hat{i} + (3x^2 - z)\hat{j} + (3xz^2 - y)\hat{k}$ is irrotational. Find ϕ such that $\overline{F} = \nabla\phi$.

(OR)

9. a) Find Curl (Curl $\overline{F}$) where $\overline{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.

b) If ϕ is a scalar point function and $\overline{F}$ is a vector point function then prove that $div(\phi\overline{F}) = \phi div\overline{F} + grad\phi.\overline{F}$.

PART - D

Answer two full questions. (2×10=20)

10. a) Use the method of separation of symbols to prove that

$$U_0+U_1x+U_2x^2+......+\infty=\frac{U_0}{1-x}+\frac{xU_0}{(1-x)^2}+\frac{x^2\Delta^2U_0}{(1-x)^3}+.....\infty.$$

b) Obtain a function whose first difference is $x^3+3x^2+5x+12$.

(OR)

11. a) Find a cubic polynomial which takes the following data.

x	0	1	2	3
f(x)	1	2	1	10

b) Find f(9.7) from the following data

x	8	8.5	9	9.5	10
f(x)	50	57	64	71	75

12. a) Using Lagranges interpolation formula find f(2) from the following data

x	0	1	3	4
f(x)	5	6	50	105

b) Using Simpson's $\frac{3}{8}^{th}$ rule Evaluate $\int_0^6 \frac{1}{1+x^2}dx$.

(OR)

13. a) Evaluate $\int_0^{0.6} e^{-x^2}dx$ by taking 6 subintervals By using Simpson's $\frac{1}{3}^{rd}$ Rule.

b) Prepare divide difference table for the following data.

x	1	3	4	6	10
f(x)	0	18	58	190	920

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V Semester B.Sc. Degree Examination, April - 2022

MATHEMATICS

(CBCS Semester Scheme)

Paper : VI

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

PART - A

Answer any **five** questions. **(5×2=10)**

1. a) Write Euler's equation, when f is independent of y.

   b) Show that the functional $I = \int_{x_1}^{x_2} (y^2 + x^2 y^1) dx$ assumes extreme values on the straight line y = x.

   c) Define geodesic on a surface.

   d) Evaluate $\int_C [(3x+y)dx + (2y-x)dy]$ along $y = x$ from (0,0) to (10,10).

   e) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^a r^2 dr d\theta$.

   f) Evaluate $\int_0^1 \int_0^2 \int_0^3 (x+y+z) dxdydz$.

   g) State Gauss - Divergence theorem.

   h) Evaluate $\oint_C (yzdx + zxdy + xydz)$ where 'C' is the curve $x^2 + y^2 = 1$, $z = y^2$ using Stoke's theorem.

PART - B

Answer **two** full questions. **(2×10=20)**

2. a) Derive Euler's equation in the form $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y^1}\right) = 0$.

   b) Prove that geodesic on a plane is a straight line.

(OR)

[P.T.O.

3. a) Solve the variational problem $\delta\int_0^{\frac{\pi}{2}}\left[y^2-(y')^2\right]dx=0$ under the conditions $y(0)=0,\ y\left(\frac{\pi}{2}\right)=2$.

b) Show that the extremal of $\int_{x_1}^{x_2}\left(\frac{y'}{y}\right)^2 dx$ can be expressed in the form $y=ae^{bx}$.

4. a) Prove that catenary is the curve which when rotated about a line generates a surface of minimum area.

b) Find the extremal of the functional $\int_{x_1}^{x_2}\left[12xy+(y')^2\right]dx$.

(OR)

5. a) Find the extremal of the functional $I=\int_0^1\left[(y')^2+x^2\right]dx$ subject to the constraint $\int_0^1 ydx=2$ and having end conditions $y(0)=0, y(1)=1$.

b) Find the function y which makes the integral $I=\int_{x_1}^{x_2}[y^2+4(y')^2]dx$ an extremum.

PART - C

Answer **two** full questions. **(2×10=20)**

6. a) Evaluate $\int_C[(x+2y)dx+(4-2x)dy]$ along the curve $C:\frac{x^2}{16}+\frac{y^2}{9}=1$ in anticlockwise direction.

b) Evaluate $\iint_A(4x^2-y^2)dxdy$, where A is the area bounded by the lines y = 0, y = x and x = 1.

(OR)

7. a) Evaluate $\int_0^a\int_0^{2\sqrt{ax}}x^2dxdy$ by changing the order of integration.

b) Compute $\int_C(xdx+ydy)$ around the square (0,0), (1,0), (1,1), (0,1).

8. a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx$.

b) Evaluate $\iint_R \frac{x^2 y^2}{x^2+y^2} dx\, dy$ by changing into polar coordinates, where R is the annular region between the circles $x^2+y^2=4$ and $x^2+y^2=1$.

(OR)

9. a) Evaluate $\iiint_V xy\, dx\, dy\, dz$ over the region bounded by the coordinate planes and the plane $x+y+z=1$.

b) Find volume of the sphere $x^2+y^2+z^2=a^2$ by triple integration.

PART - D

Answer **two** full questions. **(2×10=20)**

10. a) State and prove Green's theorem.

b) Evaluate using Gauss - Divergence thorem $\iint_S (\overline{F}.\hat{n})\, dS$, where $\overline{F}=(2xy)\hat{i}+(yz^2)\hat{j}+(xz)\hat{k}$ and S is the total surface of the rectangular parallelopiped bounded by the planes x = 0, y = 0, z = 0, x =1, y = 2, z = 3.

(OR)

11. a) Evaluate by Stoke's theorem

$\int_C (\sin z dx - \cos x dy + \sin y dz)$, where 'C' is the boundary of the rectangle $0 \le x \le \pi, 0 \le y \le 1$, z=3.

b) Using Green's theorem, evaluate $\oint_C (y^2 dx + x^2 dy)$, where C is the closed curve bounded by y = x and y^2=x.

12. a) Using Gauss - Divergence theorem, evaluate $\iint_S \left(x\hat{i}+y\hat{j}+z\hat{k}\right).\hat{n}\, dS$, where S is closed surface bounded by the cone $x^2+y^2=z^2$ and the plane z=1.

b) Using stoke's theorem, evaluate $\oint_C \overline{F}.d\overline{r}$ where $\overline{F}=(2x-y)\hat{i}-(yz^2)\hat{j}-(y^2z)\hat{k}$ and S is the upper half of the sphere $x^2+y^2+z^2=1$ and C is its boundary.

(OR)

[P.T.O.

13. a) Using Gauss - Divergence theorem, evaluate $\iint_S (\vec{F}.\hat{n})dS$, where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$ over the rectangular parallelepiped $0 \le x \le a,\ 0 \le y \le b, 0 \le z \le c$.

b) Using Green's theorem, evaluate $\int_C [(xy + y^2)dx + x^2dy]$, where 'C' is the closed curve bounded by y = x and $y = x^2$.