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V Semester B.A./B.Sc. Examination, December - 2019
(CBCS) (F+R) 2016-17 and Onwards)

MATHEMATICS - V

Time : 3 Hours Max. Marks : 70

Instruction : *Answer* ***all*** *questions.*

PART - A

Answer **any five** questions. **5x2=10**

1. (a) Give an example of

(i) a ring with zero divisor

(ii) a non-commutative ring with unity

(b) In a ring $(R, +, \cdot)$ prove that $a\cdot(b-c)=a\cdot b-a\cdot c\ \forall a, b, c \in R$.

(c) Define principal and maximal ideals of a ring R.

(d) Find the maximum directional derivative of $\phi=x^3y^2z$ at the point $(1, -2, 3)$.

(e) If $\vec{f}=3x^2\hat{i}+5xy^2\hat{j}+xyz^3\hat{k}$ then, find $\operatorname{div}\vec{f}$ at $(1, 2, 3)$.

(f) Evaluate : $\Delta^4(1-ax)(1-bx)(1-cx)(1-dx)$.

(g) Write Lagrange's Interpolation formula for unequal intervals.

(h) Using Trapezoidal rule, evaluate $\int_0^6 f(x)\,dx$ given :

x	0	1	2	3	4	5	6
$f(x)$	0.146	0.161	0.176	0.190	0.204	0.217	0.230

PART - B

Answer **two** full questions. **2x10=20**

2. (a) Prove that the set $R=\{0, 1, 2, 3, 4, 5\}$ is a commutative ring w.r.t. $\oplus_6$ and $\otimes_6$ as two compositions.

(b) Prove that a ring R is without zero divisors if and only if the cancellation laws hold in it.

OR

P.T.O.

3. (a) Prove that the necessary and sufficient conditions for a non-empty subset S to be a subring of R, are :
(i) $S+(-S)=S$ (ii) $SS \subseteq S$

(b) Define the right and left ideals of a ring R. Show that the subset

$$S=\left\{\begin{pmatrix} a & 0 \\ b & 0 \end{pmatrix} \middle| a, b \in z\right\}$$ of $M_2(z)$ is a left ideal but not a right ideals of $M_2(z)$.

4. (a) (i) If 'a' is an element of a commutative ring R, then prove that $aR=\{ar \mid r \in R\}$ is an ideal of 'R'.
(ii) If I is an ideal of a ring 'R' with unity and $1 \in I$ then prove that $I=R$.

(b) Find all the principal ideals of the ring $R=\{0, 1, 2, 3, 4, 5\}$ w.r.t. $\oplus_6$ and $\otimes_6$ as two compositions.

OR

5. (a) If $f: R \to R'$ is a homomorphism of a ring R into R' then prove that
(i) $f(0)=0'$ where 0 and 0' are the zero elements of R and R' respectively.
(ii) $f(-a)=-f(a)\ \forall a \in R$.

(b) State and prove fundamental theorem of homomorphism of rings.

PART - C

Answer **two** full questions. **2x10=20**

6. (a) Find the constants a and b so that the surfaces $x^2+ayz=3x$ and $bx^2y+z^3=(b-8)y$ intersect orthogonally at the point $(1, 1, -2)$.

(b) If ϕ is a scalar point function and $\vec{f}$ is a vector point function then

prove that $\operatorname{div}(\phi \vec{f}) = \phi(\operatorname{div} \vec{f}) + (\operatorname{grad}\phi) \cdot \vec{f}$

OR

7. (a) If $\vec{r} = x\hat{i}+y\hat{j}+z\hat{k}$ then prove that $\nabla^2(r^3 \vec{r})=18r\vec{r}$ where $r=|\vec{r}|$

(b) If $\vec{F}=\nabla\left(x^3+y^3+z^3-3xyz\right)$ then find $\nabla \cdot \vec{F}$ and $\nabla \times \vec{F}$

8. (a) Show that $\operatorname{div}(\vec{a} \times (\vec{r} \times \vec{a}))=2|\vec{a}|^2$ where $\vec{a}$ is a constant vector.

(b) Show that $\vec{F}=(x^2-yz)\hat{i}+(y^2-xz)\hat{j}+(z^2-xy)\hat{k}$ is irrotational. Also find a scalar function ϕ such that $\vec{F}=\nabla\phi$

OR

9. (a) (i) If $\vec{F} = 3xy\hat{i} + 20yz^2\hat{j} - 15xz\hat{k}$ and $\phi = xyz$, then find $\text{curl}(\phi\vec{F})$.

(ii) Show that $\vec{F} = 2x^2z\hat{i} - 10xyz\hat{j} + 3xz^2\hat{k}$ is solenoidal.

(b) Find $\text{curl}(\text{curl}\vec{F})$ if $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.

PART - D

Answer **two** full questions. **2x10=20**

10. (a) Use the method of separation of symbols to prove that

$$u_0 - u_1 + u_2 - u_3 + \ldots.. + \infty = \frac{1}{2}u_0 - \frac{1}{4}\Delta u_0 + \frac{1}{8}\Delta^2 u_0 - \frac{1}{16}\Delta^3 u_0 + \ldots..$$

(b) Obtain a function whose first difference is $x^3 + 3x^2 + 5x + 12$.

OR

11. (a) Find the number of students from the following data who secured marks not more than 45.

Marks	30 - 40	40 - 50	50 - 60	60 - 70	70 - 80
Number of Students	35	48	70	40	22

(b) Find a polynomial of lowest degree which assumes the values 10, 4, 40, 424, 620 at $x = -2$, 1, 3, 7 and 8 respectively, using Newton's divided difference formula.

12. (a) By employing Newton-Gregory backward difference formula, find $f(9.7)$ from the following data.

x	8	8.5	9	9.5	10
$f(x)$	50	57	64	71	75

(b) Using Simpson's $\frac{1}{3}^{rd}$ rule, Evaluate $\int_0^1 \frac{1}{1+x^2}dx$ dividing the interval (0, 1) into 8 equal parts.

OR

13. (a) Applying Lagrange's formula find $f(5)$, given that $f(1)=2$, $f(2)=4$, $f(3)=8$ and $f(7)=128$.

(b) Using Simpson's $\frac{3}{8}^{th}$ rule, Evaluate $\int_4^{5.2} \log_e x\,dx$ taking $h=0.2$.

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V Semester B.A./B.Sc. Examination, December - 2019

(CBCS) (F+R) (2016-17 and Onwards)

MATHEMATICS - VI

Time : 3 Hours | Max. Marks : 70

Instruction : *Answer **all** questions.*

PART - A

Answer **any five** questions. **5x2=10**

1. (a) Write Euler's equation when the function 'f' is independent of x and y.

(b) Find the curve $\int_0^1 \left[12xy+(y')^2\right] dx=0$ with $y(0)=3$, $y(1)=6$.

(c) Find the function y which makes the integral $I=\int_{x_1}^{x_2}\left[1+xy'+x(y')^2\right] dx$ an extremum.

(d) Evaluate $\int_C x\,dy - y\,dx$, where 'C' is a line $y=x^2$ from (0, 0) to (1, 1).

(e) Evaluate $\int_0^2 \int_0^1 (x+y)\,dx\,dy$

(f) Evaluate $\int_0^1 \int_0^1 \int_0^1 \left(x^2+y^2+z^2\right) dx\,dy\,dz$

(g) State Gauss Divergence Theorem.

(h) Write vector form of Green's Theorem.

P.T.O.

PART - B

Answer **two full** questions. **2x10=20**

2. (a) Prove necessary condition for the integral $I=\int_{x_1}^{x_2} f(x,y,y')\,dx$, where $y(x_1)=y_1$ and $y(x_2)=y_2$ to be an extremum that $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$.

   (b) Find the extremal of the functional $I=\int_{x_1}^{x_2}\left(y^2+y'^2+2ye^x\right)dx$.

OR

3. (a) Show that an extremal of $\int_{x_1}^{x_2}\left(\frac{y'}{y}\right)^2 dx$ is expressible in the form $y=ae^{bx}$.

   (b) Solve the variational problem $\delta\int_1^2\left[x^2y'^2+2y(x+y)\right]dx=0$ with the conditions $y(1)=0=y(2)$.

4. (a) Find the shape of a chain which hangs under gravity between two fixed points.

   (b) Find the extremal of the functional $I=\int_0^{\pi}\left(y'^2-y^2\right)dx$ under the conditions $y=0,\ x=0,\ x=\pi,\ y=1$ subject to the condition $\int_0^{\pi} y\,dx=1$.

OR

5. (a) Find the extremal of the functional $\int_0^1\left(x+y+y'^2\right)dx=0$ under the conditions $y(0)=1$ and $y(1)=2$.

   (b) Find the geodesic on a right circular cylinder.

PART - C

Answer **two full** questions. **2x10=20**

6. (a) Evaluate $\int_C (x+2y)dx+(4-2x)\,dy$ along the curve C : $\frac{x^2}{16}+\frac{y^2}{9}=1$ in anticlockwise direction.

(b) Evaluate $\iint_R xy\,dx\,dy$ over the positive quadrant bounded by the circle $x^2+y^2=1$.

OR

7. (a) Evaluate $\int_C (x+y+z)ds$, where 'C' is the line joining the points (1, 2, 3) and (4, 5, 6) whose equations are $x=3t+1$, $y=3t+2$, $z=3t+3$.

(b) Change the order of integration and hence evaluate $\int_0^a \int_0^{2\sqrt{ax}} x^2\,dx\,dy$.

8. (a) Find the area $\iint_S \frac{y}{x}e^x\,dx\,dy$, where S is bounded by $x=y^2$ and $y=x^2$.

(b) Find the volume of the tetrahedron by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=a$.

OR

9. (a) Change into polar co-ordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dx\,dy$.

(b) If R is the region bounded by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=1$. Show that $\iiint_R z\,dx\,dy\,dz=\frac{1}{24}$.

PART - D

Answer **two full** questions. 2x10=20

10. (a) State and prove Green's theorem.

(b) Evaluate by Stoke's Theorem $\oint_C (yz\,dx + xz\,dy + xy\,dz)$, where C is the curve $x^2 + y^2 = 1$, $z = y^2$.

OR

11. (a) Verify Green's theorem $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where 'C' is the region bounded by parabolas $y^2 = x$ and $x^2 = y$.

(b) Using divergence theorem, show that :

(i) $\iint_S \vec{r} \cdot \hat{n}\,ds = 3V$ and

(ii) $\iint_S (\nabla r^2) \cdot \hat{n}\,ds = 6V$

12. (a) Evaluate using Gauss' divergence theorem $\iint_S \vec{F} \cdot \hat{n}\,ds$, where $\vec{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the total surface of the rectangular parallelopiped bounded by the planes $x = 0$, $y = 0$, $z = 0$, $x = 1$, $y = 2$, $z = 3$.

(b) Evaluate $\iint_S \left(\mathrm{Curl}\,\vec{F}\right) . \hat{n}\,ds$ by Stoke's theorem, where $\vec{F} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$ and S is the surface of the cube $0 \le x \le 2$, $0 \le y \le 2$, $0 \le z \le 2$.

OR

13. (a) Evaluate $\iint_S \vec{F} \cdot \hat{n}\,ds$ using divergence theorem, where $\vec{F} = (x^2 - yz)\hat{i} + (y^2 - xz)\hat{j} + (z^2 - xy)\hat{k}$ taken over rectangular box $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

(b) Evaluate by Stoke's theorem $\oint_C (\sin z\,dx - \cos x\,dy + \sin y\,dz)$, where C is the boundary of rectangle $0 \le x \le \pi$, $0 \le y \le 1$, $z = 3$.

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