11525

Reg. No. | | | | | | | | |

V Semester B.Sc. Degree Examination, March - 2021

MATHEMATICS - 5

(CBCS Semester Scheme)

Paper : V

M.E.S. COLLEGE LIBRARY, BENGALURU

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer **ALL** Questions.

PART - A

Answer any **Five** questions. **(5×2=10)**

1. a) In a ring $(R,+,\cdot)$, show that $a.0=0.\ a=0\ \forall a\in R$.

   b) Define a field. Give an example.

   c) Prove that every ideal of a ring R is a subring of R.

   d) Find the unit normal vector to the surface $3x^2+2y^2+4z^2=9$ at (1,-1,1).

   e) Show that $\phi=x^2-y^2+4z$ is a harmonic function.

   f) Prove that $E\ \nabla=\nabla E$.

   g) Write Lagrange's interpolation formula.

   h) Evaluate $\int_0^3 \frac{dx}{1+x}$ by Simpson's $1/3^{rd}$ rule by dividing the interval into two equal parts.

PART - B

Answer **Two** full questions. **(2×10=20)**

2. a. Prove that the set $R=\{0,1,2,3,4,5,6\}$ is a commulative ring with respect to

   $\oplus_7$ and $\otimes_7$.

   b. Prove that every field is an integral domain.

[P.T.O.

(OR)

3. a. If p is an integer then pz is maximal ideal of $(z,+,\cdot)$ if and only if p is prime.

b. Find all the principal ideals of the ring $R=\{0,1,2,3,4,5\}$ with respect to $\oplus_6$ and $\otimes_6$.

4. a. If $f:R\rightarrow R'$ is a homomorphism then

Prove that

i) f(R) is a a subring of R'

ii) Ker(f) is a subring of R.

b. If $f:R\rightarrow R'$ is a homomorphism of rings R and R' with Kernel k, then prove that f is one - one if and only if k=\{0\}.

(OR)

5. a. Prove that the mapping $f:Z\rightarrow S$ where Z is the ring of integers and ring $S=\left\{\begin{pmatrix} a & 0 \\ 0 & 0\end{pmatrix} / a\in Z\right\}$, defined by $f(a)=\begin{pmatrix} a & 0 \\ 0 & 0\end{pmatrix}$ $\forall a\in Z$. Show that f is an isomorphism.

b. Prove that every homomorphic image of a ring R is isomorphic to some residue class ring thereof.

PART - C

Answer any **Two** questions (full) : **(2×10=20)**

6. a. Find the directional derivative of $\phi(x,y,z)=x^2-y^2+4z^2$ at the point (1,1,-8) in the direction of 2i+j-k.

b. If $\vec{r}=xi+yj+zk$ and $r=|\vec{r}|$, prove that $\nabla r^n=nr^{n-2}\vec{r}$.

(OR)

7. a. Show that $\vec{F}=(2xy^2+yz)i+(2x^2y+xz+2yz^2)j+(2y^2z+xy)k$ is irrotational. Also find a scalar function ϕ such that $\vec{F}=\nabla\phi$.

b. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $x^2+y^2-z=3$ at the point (2,-1,2).

8. a. Find the constants a,b,c so that the vector
$\vec{F}=(x+2y+az)i+(bx-3y-z)j+(4x+cy+2z)k$ is irrotational.

b. Prove that

i. $Curl(grad\phi)=\vec{0}$

ii. $div(curl\vec{F})=0$.

(OR)

9. a. Prove that $\nabla^2 r^n = n(n+1)r^{n-2}$, where n is a non - zero constant. Also prove that r^n is harmonic if $n=-1$.

b. If ϕ is a scalar function and $\vec{F}$ is a vector function then prove that $div(\phi\vec{F})=\phi(div\vec{F})+(grad\phi).\vec{F}$.

PART - D

Answer any **Two** full questions. **(2×10=20)**

10. a. Find the second degree polynomial which takes the following values

x	1	2	3	4
y	-1	-1	1	5

b. From the following data find θ at $x = 84$

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

(OR)

11. a. Using Lagrange's interpolation formula find f(6) from the following data

x	3	7	9	10
f(x)	168	120	72	63

b. Using Newton divided difference formula find f(10) from the following data.

x	4	7	9	12
f(x)	-43	83	327	1053

12. a. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Trapezoidal rule.

b. Find f(7.5) from the following data

x	7	8	9	10
$f(x)$	3	1	1	9

(OR)

13. a. Evaluate $\Delta(e^{3x}\log 4x)$.

b. Using Simpson's $\frac{1}{3}^{rd}$ rule find $\int_0^{0.6} e^{-x^2}\,dx$ from the following table.

x	0	0.1	0.2	0.3	0.4	0.5	0.6
$y=e^{-x^2}$	1	0.99	0.9608	0.9131	0.8521	0.7788	0.6977

11526

Reg. No.

V Semester B.Sc. Degree Examination, March - 2021
MATHEMATICS
(CBCS Semester Scheme)
Paper : VI

M.E.S. COLLEGE LIBRARY

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer All questions.

PART - A

Answer any **Five** questions. **(5×2=10)**

1. a) Write Euler's equation when f is independent of y.

b) Find the differential equation of the functional $I = \int_{x_1}^{x_2} \left[xy' - (y')^2\right] dx$.

c) Define geodesic on a surface.

d) Evaluate $\int_c x\,dx + y\,dy$ along the parabola $y^2 = x$ from (1,1) to (2,2).

e) Evaluate $\int_0^1 \int_0^{1-x} x^2 y\, dy\, dx$

f) Evaluate $\int_0^3 \int_0^2 \int_0^1 xyz\, dx\, dy\, dz$.

g) State stokes theorem.

h) If $\vec{F} = curl\,\overline{A}$ then prove that $\int\int_s \vec{F}.\hat{n}\, ds = 0$ for any closed surface using Gauss divergence theorem.

[P.T.O.

PART - B

Answer Two full questions. (2×10=20)

2. a) Find the extremal of the functional $I=\int_{x_1}^{x_2}\left[12xy+(y')^2\right]dx$.

b) Prove that a necessary condition for the functional $I=\int_{x_1}^{x_2} f(x,y,y')\,dx$ to be an extremum is $\frac{\partial f}{\partial y}-\frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right)=0$, Where $y(x_1)=y_1,\ y(x_2)=y_2$.

(OR)

3. a) Find the Extremal of the functional $I=\int_0^1 \sqrt{(y')^2+1}\,dx$ given $y(0)=1$ and $y(1)=2$.

b) Find the geodesic on a plane.

4. a) Find the Extremal of the functional $I=\int_0^1 (y')^2\,dx$ subject to the constraint $\int_0^1 y\,dx=1$ and having $y(0)=0$ and $y(1)=1$.

b) Find the function y which makes the integral $I=\int_{x_1}^{x_2}\left[1+xy'+x(y')^2\right]dx$ an extremum.

(OR)

5. a) If a cable hangs freely under gravity from two fixed points. Show that the shape of the cable is a catenary ie $y=C\cosh\left(\frac{x+a}{c}\right)$.

b) Find the Extremal of the functional $I=\int_0^4\left[xy'-(y')^2\right]dx$ given y(0)=0, y(4)=3.

PART - C

Answer **Two** full questions. **(2×10=20)**

6. a) Evaluate $\int_C (3x-2y)dx+(y+2z)dy-x^2dz$ where C is the curve given by x = t. y=2t, z=3t and $0 \le t \le 1$.

b) Evaluate $\iint_D xy(x+y)\,dx\,dy$ over the domain D between $y=x^2$ and $y=x$.

(OR)

7. a) Compute $\int_C xdx+ydy$ around the square (0,0),(1,0),(1,1),(0,1).

b) Evaluate by changing the order of integration $\int_{x=0}^{1}\int_{y=0}^{\sqrt{1-x^2}} y^2\,dy\,dx$.

8. a) Evaluate $\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$.

b) Find the volume bounded by the surface $z=a^2-x^2$ and the plane x = 0, y = 0, z = 0 and y = b.

(OR)

9. a) Evaluate $\iiint_V xy\,dx\,dy\,dz$ over the region bounded by co-ordinate planes and the plane $x+y+z=1$.

b) Find the area of the circle $x^2+y^2=a^2$ by double integration.

PART - D

Answer **Two** full questions. **(2×10=20)**

10. a) State and prove Green's theorem in the plane.

b) Using Gauss divergence theorem, Evaluate $\iint_S \overline{F}\,\hat{n}\,d\,s$ where $\overline{F} = 2xy\hat{i} + yz^2\hat{j} + xz\hat{k}$ and S is the total surface of the rectangular parallelepiped bounded by the planes $x = 0$ $y = 0$, $z = 0$, $x = 1$, $y = 2$, $z = 3$.

(OR)

11. a) Using Green's theorem evaluate $\oint_C y^2dx + x^2dy$ where C is the closed curve bounded by $y = x$ and $y^2 = x$.

b) Using divergence theorem evaluate $\iint_S \overline{F}.\hat{n}\,ds$ *where* $\overline{F} = x^3\hat{i} + y^3\hat{j} + z^3\hat{k}$ *and S* is the surface of the sphere $x^2 + y^2 + z^2 = a^2$.

12. a) Using stokes theorem evaluate $\oint_C \overline{F}.d\overline{r}$ where $\overline{F} = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

b) Evaluate by using Green's theorem $\int_C (3x^2 - 4y^2)dx + (6xy)dy$ where C is the boundary of the region enclosed by the lines $x = 0$, $y = 0$, $x+y = 1$.

(OR)

13. a) Evaluate by using Gauss divergence theorem for $\overline{F} = (x^2 - yz)\hat{i} - 2x^2y\hat{j} + z\hat{k}$ taken over the cube $0 \le x \le 1$, $0 \le y \le 1$, $0 \le z \le 1$.

b) Evaluate using by stokes theorem $\oint_C yz\,dx + zx\,dy + xy\,dz$ where C is the curve $x^2 + y^2 = 1$, $z = y^2$.