12525

Reg. No. | | | | | | | | |

V Semester B.Sc. Degree Examination, March/April - 2023

MATHEMATICS

(CBCS Semester Scheme)

Paper : 5

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

I. Answer any FIVE questions. **(5×2=10)**

1. Define commutative ring and ring with zero divisors.
2. Define left ideal and right ideal of a ring (R, +, .)
3. Define isomorphism of rings.
4. What is a geodesic on a plane.
5. If u, v are functions of x, y, y'. Prove that $\delta(uv) = u\delta v + v\delta u$.
6. Prove that $\delta = E^{\frac{1}{2}}\nabla$.
7. Write Newton backward interpolation formula.
8. Write Lagrange's inverse interpolation formula for unequal intervals.

II. Answer any Two questions. **(2×5=10)**

9. Prove that $(z_5, +_5, \times_5)$ is a commutative ring with respect to addition and multiplication modulo 5.
10. Prove that the necessary and sufficient condition for a non empty subset of a ring R to be a subring are
    i) $a - b \in S$
    ii) $ab \in S \quad \forall a, b \in S$
11. Prove that the intersection of two ideals is an ideal.

[P.T.O.

III. **Answer any Two questions.** **(2×5=10)**

12. State and prove Euler's formula to find the extremum of the functional $\int_{x_1}^{x_2} f(x,y,y')dx$.

13. Show that the extremum of a functional $\int_{x_1}^{x_2} \sqrt{y(1+y'^2)}\,dx$ is a parabola.

14. Find the extremal of the functional $\int_0^1 y'^2\,dx$ subject to $\int_0^1 y\,dx = 1$ and having the constraints $y(0)=0, y(1)=1$ is a parabolic curve.

IV. **Answer any Three questions.** **(3×5=15)**

15. Prove that i) $(1+\Delta)(1-\nabla)=1$

ii) $\frac{\Delta}{\nabla}-\frac{\nabla}{\Delta}=\Delta+\nabla$

16. Find the polynomial of degree 2 for the following data.

x	0	1	2	3	4	5	6	7
y	1	2	4	7	11	16	22	29

17. Find f(2) by using Lagrange's interpolation formula for the data

x	5	6	9	11
$f(x)$	12	13	14	16

18. Express $3x^3 + x^2 + x + 1$ in factorial notation.

19. Evaluate $\int_0^3 \frac{dx}{(1+x)^2}$ by using Simpson's $\frac{3^{th}}{8}$ rule.

V. Answer any Three questions. **(3×5=15)**

20. If $f : R \to R^1$ is an isomorphism and R is a ring with unity, then prove that R^1 is also a ring with unity.

21. Show that the extremum of the functional $I = \int_{x_1}^{x_2} \left(\frac{y^1}{y}\right)^2 dx$ is expressible in the form of $y = ae^{bx}$.

22. Find the curve which makes the functional $\int_0^1 \left(y'^2 + x\right) dx$ as an extermum. Given $y(0) = 0, y(1) = 1$ under the constraint $\int_0^1 y\,dx = 1$.

23. Obtain the function whose first difference is $6x^2 + 10x + 11$.

24. Find $\int_1^5 \log_{10} x\,dx$ by taking 8 sub intervals and correct to four decimal places by using trapezoidal rule.

VI. Answer any Two questions. **(2×5=10)**

25. Find the plane curve of length l, having end points at (x_1, y_1) and (x_2, y_2) such that the area under the curve (between $x = x_1$, and $x = x_2$) is maximum.

26. The velocity v(meter/sec) of a particle at a distance s(meters) from a point on its linear path is given by the following table.

distance (s)	0	2.5	5	7.5	10	12.5	15
Velocity v)	16	19	21	22	20	17	13

Estimate the time taken by the particle to traverse the distance of 15m using Simpson's $\frac{3^{th}}{8}$ rule.

27. A reservoir discharging water through sluices at a depth 'x' below water surface has water surface area A for various values of x is as given in table. If 't' denotes the time in minutes, the rate of fall of surface is given by $\frac{dx}{dt} = -48\left(\frac{x}{A}\right)$. Estimate the time taken for the water level to fall from 14 to 10 feet above the sluices using Simpson's $\frac{3^{th}}{8}$ rule.

x feet	10	11	12	13	14
A square feet	950	1070	1200	1350	1530

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V Semester B.Sc. Degree Examination, March/April - 2023

MATHEMATICS

(CBCS Semester Scheme 2020 and Onwards)

Paper : VI

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

Answer all questions.

I. **Answer any FIVE questions.** **(5×2=10)**

1. Find $\nabla^2\phi$ for $\phi = x^2y^2z^2$ at (1, 2, 3).

2. If $\vec{F} = yz\hat{i} + zx\hat{j} + xy\hat{k}$ then show that $\vec{F}$ is irrotational.

3. Evaluate $\int_C 5x\,dx + y\,dy$ where C is the curve $y = 2x^2$ from (0, 0) to (1, 2).

4. Evaluate $\int_0^1 \int_0^{1-x} dy\,dx$.

5. Evaluate $\int_0^3 \int_0^2 \int_0^1 xyz\,dx\,dy\,dz$.

6. Evaluate $\int_0^{\pi/2} \int_0^a r^2\,dr\,d\theta$.

7. State Gauss Divergence theorem.

8. Using Stoke's theorem, evaluate $\oint_C \vec{r}.d\vec{r}$

[P.T.O.

II. Answer any Two questions. (2×5=10)

9. Find constants a, b, c if the vector

$\vec{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational.

10. Find the directional derivative of $\phi(x, y, z) = x^2 - y^2 + 4z^2$ at $(1, 1, -8)$ in the direction of $2\hat{i} + \hat{j} - \hat{k}$.

11. If $r^2 = x^2 + y^2 + z^2$, prove that $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$.

III. Answer any Three questions. (3×5=15)

12. Evaluate $\iint xy\,dx\,dy$ over the positive quadrant bounded by the circle $x^2+y^2=1$.

13. Show that the line Integral $\int_C (x^3 + 2yz)dy + 3x^2 y dx + y^2 dz$ is independent of path joining the points $(1, -2, 3)$ and $(3, 2, -1)$ and hence evaluate.

14. Evaluate $\int_0^1 \int_{y^2}^{\sqrt{y}} xy\,dx\,dy$ by changing the order of integration.

15. Find the volume underneath the surface $x + y + z = 2$ which cuts the cylinder $x^2 + y^2 = 1$ above the XY plane.

16. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{1}{\sqrt{1 - x^2 - y^2 - z^2}} dz\,dy\,dx$.

IV. Answer any Two questions. (2×5=10)

17. Using Green's theorem, evaluate $\int_C y^2 dx + x^2 dy$ where C is the closed curve bounded by $y = x$ and $y^2 = x$.

18. Using Gauss Divergence theorem, evaluate $\iint_S \vec{F}\,\hat{n}\,ds$ where $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ and S is the surface enclosing the region for which $x^2 + y^2 \leq 4$ and $0 \leq z \leq 3$.

19. Evaluate by using Stoke's theorem $\oint_C \sin z\,dx - \cos x\,dy + \sin y\,dz$ where C is the boundary of the rectangle $0 \leq x \leq \pi$, $0 \leq y \leq 1$, $z = 3$ and its boundary.

V. **Answer any Three questions.** **(3×5=15)**

20. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = |\vec{r}|$, then prove that

a) $\nabla r^n = n r^{n-2}\vec{r}$

b) $\nabla\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$

21. If ϕ is a scalar function, then prove that curl (grad ϕ) = 0.

22. Evaluate $\iint_R \frac{x^2 y^2}{x^2 + y^2} dx\,dy$ by transforming to polar co-ordinates where R is the region between the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

23. Find the volume of the cylinder $x^2 + y^2 = a^2$ bounded by the planes $x = 0, y = 0, z = 0$, $z = b$ by transforming to cylindrical polar co-ordinates.

24. Using Green's theorem, find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

VI. **Answer any Two questions.** **(2×5=10)**

25. Find the work done in moving a particle under a force $\vec{F} = 2xy\hat{i} - 3x\hat{j} - 5z\hat{k}$ a long the curve $x = t$, $y = t^2 + 1$, $z = 2t^2$ from $t = 0$ to $t = 1$.

26. Find the moment of inertia of a circular plate of mass 'm' of radius 'a' about the x-axis through its centre.

27. Determine the x co-ordinate of centre of gravity of the plane lamina of uniform surface density bounded by upper half of the cardiod $r = a(1 + \cos\theta)$.