12625

Reg. No. | | | | | | | | |

VI Semester B.Sc. Degree Examination, August/September - 2023

MATHEMATICS

(CBCS Scheme)

Paper : VII

Time : 3 Hours

Maximum Marks : 70

Instructions to Candidates :

Answer **ALL** questions.

I. Answer any **FIVE** questions. (5×2=10)

1. Prove that the set $S=\{(3, 2, -1), (0, 4, 5), (6, 4, -2)\}$ is linearly dependent in $V_3(R)$.

2. Find the matrix of linear transformation $T: V_2(R) \to V_2(R)$ defined by $T(x, y)=(x, -y)$ with respect to the standard bases.

3. Write the relation between the cartesian Co-ordinates and cylindrical Co-ordinates of a point.

4. Prove that in spherical Co-ordinate system $\hat{e}_r \times \hat{e}_\theta = \hat{e}_\phi$.

5. Verify the integrability condition for $(y+z)dx+(x+z)dy+(x+y)dz=0$.

6. Solve $\frac{xdx}{y^2z}=\frac{dy}{zx}=\frac{dz}{y^2}$.

7. Form the partial differential equation by eliminating the arbitrary function 'f' from $z=f\left(x^2-y^2\right)$.

8. Solve $\left(D^2-4DD'+4D'^2\right)z=0$.

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II. Answer any **Three** questions. (3×5=15)

9. Prove that "The intersection of any two subspaces of a vector space V(F) is also a sub-space of V(F).

10. Find the dimension and basis of the subspace spanned by the vectors $\{(2, 4, 2), (1, -1, 0), (1, 2, 1), (0, 3, 1)\}$ of $V_3(R)$.

11. Find the matrix of linear transformation $T: R^2 \to R^2$ defined by $T(x, y) = (x + 4y, 2x - 3y)$ relative to the bases $B_1 = \{(1, 0), (0, 1)\}$, $B_2 = \{(1, 3), (2, 5)\}$.

12. State and prove rank-nullity theorem.

13. Show that the set of all eigen vectors associated with the eigen value λ of a linear transformation T together with zero vectors is a subspace of the vectorspace.

III. Answer any **Three** questions. (3×5=15)

14. Show that the spherical Co-ordinate system is orthogonal curvilinear co-ordinate system.

15. Express the vector $\vec{f} = 2x\hat{i} - 2y^2\hat{j} + xz\hat{k}$ in cylindrical Co-ordinates and find f_ρ, f_ϕ, f_z

16. Express $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in spherical polar Co-ordinates and hence find f_r, f_θ, f_ϕ.

17. Solve $\dfrac{\partial u}{\partial t} = 16\dfrac{\partial^2 u}{\partial x^2}$ given

$u(0, t) = 0,\ u(1, t) = 0,\ \forall t$

$u(x, 0) = x^2 - x,\ 0 \le x \le 1$

18. Solve

$\dfrac{\partial^2 u}{\partial t^2} = c^2 \dfrac{\partial^2 u}{\partial x^2}$ given $u(0, t) = 0,\ u(l, t) = 0,\ u(x, 0) = a\,\mathrm{Sin}\left(\dfrac{\pi x}{l}\right)$ and $\left(\dfrac{\partial u}{\partial t}\right)_{t=0} = 0$

IV. Answer any **Three** questions. (3×5=15)

19. Verify the condition of integrability and solve $3x^2dx + 3y^2dy - \left(x^3 + y^3 + e^{2z}\right)dz = 0$.

20. Solve $\dfrac{dx}{x(y-z)} = \dfrac{dy}{y(z-x)} = \dfrac{dz}{z(x-y)}$.

21. Form the partial differential equation by eliminating arbitrary function ϕ from $lx + my + nz = \phi\left(x^2 + y^2 + z^2\right)$.

22. Solve $p^3 + q^3 = 27z$.

23. Find the complete integral of $z^2\left(p^2 + q^2 + 1\right) = 1$ by using charpit's method.

V. Answer any **Three** questions. (3×5=15)

24. Find a linear transformation $T : R^2 \to R^2$ such that $T(1, 0) = (1, 1)$ and $T(0, 1) = (-1, 2)$ prove that T maps the square with vertices $(0, 0)$, $(1, 0)$, $(1, 1)$ and $(0, 1)$ into parallelogram.

25. The vibration of an elastic string is governed by $pde \dfrac{\partial^2 u}{\partial t^2} = \dfrac{\partial^2 u}{\partial x^2}$. The length of the string is π and ends are fixed. The initial velocity is zero and the initial deflection is $u(x, 0) = 2(\text{Sin } x + \text{Sin } 3x)$. Find the deflection $u(x, t)$ of the vibrating string for $t > 0$.

26. Evaluate $\iiint_v \left(x^2 + y^2 + z^2\right) dxdydz$ where v is the sphere having centre at the region and radius equal to 'a' by changing the variable to spherical polar Co-ordinates.

27. Find the curves which satisfy the differential equation $ydx + zdy - ydy + xdz = 0$ and which lie on the plane $2x - y - z = 1$.

28. Reduce the equation $r+2s+t=0$ to canonical form.

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VI Semester B.Sc. Degree Examination, August/September - 2023

MATHEMATICS

(CBCS Scheme)

Paper : VIII

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates :

Answer **ALL** questions.

I. Answer any **FIVE** questions. **(5×2=10)**

1. Show that $\arg\left(\frac{\bar{z}}{z}\right)=\frac{\pi}{2}$ represents a line through the origin.
2. Define continuity of $f(z)$ at the point $z=z_0$.
3. Define harmonic function, give an example.
4. Verify that $f(z)=u+iv$ where $u=x^2-y^2$ and $v=2xy$ are the real and imaginary parts of $f(z)$ is an analytic function.
5. Evaluate $\int_c (\bar{z})^2 dz$ around the circle $|z|=1$.
6. Define cross ratio of four points Z_1, Z_2, Z_3, Z_4.
7. Find the real root of the equation $x^3-x-2=0$ in the interval $(1.5, 2)$ upto 2 approximations by bisection method.
8. State formula for Runge-Kutta method.

II. Answer any **Three** questions. **(3×5=15)**

9. Find the locus of the point z satisfying $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{3}$
10. Evaluate $\lim_{z\to 2e^{i\pi/4}}\left(\frac{z^2-4}{z^2+z+5}\right)$
11. Find the orthogonal trajectories of the family of curves $x^3y-xy^3=6$.

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12. State and Prove necessary condition for a function $f(z)=u(x,y)+iv(x,y)$ to be analytic.

13. Find the analytic function $f(z)=u+iv$ given that $u-v=e^{x}(\cos y-\sin y)$.

III. Answer any **Three** questions. (3×5=15)

14. Evaluate $\int_{(0,1)}^{(2,5)}(3x+y)dx+(2y-x)dy$ along $y=x^2+1$.

15. State and Prove Cauchy's integral theorem.

16. Evaluate $\int_{c}\frac{e^{z}}{z(z-2)}dz$ where c is $|z|=3$.

17. Discuss the transformation $w=\operatorname{Sin}h\,z$.

18. Prove that the bilinear transformation preserves the cross ratio of four points.

IV. Answer any **Three** questions. (3×5=15)

19. Using Newton Raphson method, find the real root of the equation $x^3+5x-11=0$ by performing 3 iterations only.

20. Solve the equations by Gauss Seidel method.

$10x+2y+z=9,\ x+10y-z=-22,\ -2x+3y+10z=22$

21. Find the largest eigen value of the matrix $\begin{bmatrix}2 & 1 & 2\\ 1 & 3 & 0\\ 2 & 0 & -4\end{bmatrix}$ by power method.

22. Using Taylor series method, solve $\frac{dy}{dx}=x^2y-1,\ y(0)=1$, find $y(0.1)$ correct to 3 decimals taking upto 3rd degree term.

23. Using Runge-Kutta method, solve $\frac{dy}{dx}=\frac{1}{x+y},\ y(0.4)=1$ at $x=0.5$ correct to 3 decimal places.

V. Answer any **Three** questions. (3×5=15)

24. A two dimensional flow field is given by $\psi = xy$. Show that flow is irrotational. Find the stream lines and potential lines.

25. Expand $\frac{1}{z+1}$ about $z = 1$, in Taylor's series.

26. Show that $u = -wy,\ v = wx,\ w = 0$ represents a possible motion of inviscid fluid. Find the stream function and sketch stream lines.

27. The concentration of salt x in a home made soap maker is given as a function of time by $\frac{dx}{dt} = 37.5 - 3.5x$. At the initial time $t = 0$, the salt concentration in the tank is $50\ g/L$. Using Runge Kutta method with a step size of $h = 1.5\min$., what is the salt concentration after 1.5 minutes.

28. A polluted lake has an initial concentration of bacteria of $10^7\ parts/m^3$ while the acceptable level $5\times10^6\ parts/m^3$. The concentration of the bacteria will reduce as fresh waters enter the lake. The differential equation that governs the concentration C of the pollutant as a function of time (in weeks) is given by $\frac{dc}{dt} + 0.06\ C = 0\ \ C(0) = 10$. Using Euler's method and a step size of 3.5 weeks, find the concentration of the pollutant after 7 weeks.