11625

Reg. No. [ ]

VI Semester B.A./B.Sc. Degree Examination, September/October - 2022

MATHEMATICS - VII

(CBCS Scheme Semester Freshers & Repeaters 2016-17, 2018-19 And Onwards)

Paper : VII

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1. Answer all questions.
2. Non - programmable scientific calculators are allowed.

PART - A

Answer any Five questions. (5×2=10)

1. a. Define a vector space over field.

   b. Prove that the set $S=\{(3,2,-1),(0,4,5),(6,4,-2)\}$ is linearly dependent in $V_3(R)$.

   c. Find the matrix of the linear transformation $T:V_2(R)\rightarrow V_2(R)$ defined by $T(x,y)=(x,-y)$ w.r..t the standard bases.

   d. Define Rank and Nullity of linear transformation.

   e. In a cylindrical coordinate system prove that $\hat{e}_\phi.\hat{e}_z=0$.

   f. Solve $\frac{dx}{yz}=\frac{dy}{zx}=\frac{dz}{xy}$.

   g. Form a partial differential equation by eliminating arbitrary constant from $z=(x-a)^2+(y-b)^2$.

   h. Solve $p^2-q^2=1$.

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PART - B

Answer two full questions. (2×10=20)

2. a. Prove that a subset W of a vector space V(F) is a subspace of V(F) if and only if $C_1\alpha + C_2\beta \in W$, for $\alpha, \beta \in W$.

   b. Find the dimension and basis of the subspace spanned by the vectors (2,-3,1), (3,0,1), (0,2,1), (1,1,1) of $V_3(R)$.

(OR)

3. a. Show that $V = \left\{ \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} \Big/ x, y \in R \right\}$ is a vector space over R.

   b. Prove that $W = \{(x, y, z) / x = y = z\}$ is a subspace of R^3.

4. a. Find the linear transformation $T: R^2 \to R^3$ such that T(1,1)=(0,1,2), T(-1,1) = (2,1,0).

   b. Find the matrix of linear transformation $T: R^2 \to R^2$ defined by $T(x, y) = (x + 4y, 2x - 3y)$ relative to bases $B_1 = \{(1,0), (0,1)\}$, $B_2 = \{(1,3), (2,5)\}$.

(OR)

5. a. Let $T: V \to W$ be a linear transformation, Then show that

   i. R(T) is a subspace of W.
   ii. N(T) is a subspace of V.
   iii. T is one - one if and only if N(T(= {0}.

   b. Find the range space, null space, rank, nullity and verify Rank - Nullity theorem for $T: V_3(R) \to V_3(R)$ defined by $T(x, y, z) = (x + y, x - y, 2x + z)$.

PART - C

Answer Two full questions. (2×10=20)

6. a. Verify the condition of integrability and solve

   $2yzdx + zxdy - xy(1 + z)dz = 0$.

   b. Solve : $(y^2 + z^2)p - xyq + xz = 0$.

(OR)

7. a. Show that spherical coordinate system is an orthogonal curvilinear coordinate system.

b. Express the vector $\vec{f} = yz\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinates and find f_ρ, f_ϕ and f_z.

8. a. Solve $\frac{dx}{1+y} = \frac{dy}{1+x} = \frac{dz}{z}$.

b. Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$.

(OR)

9. a. Express the vector $\vec{f} = 3y\hat{i} + 2z\hat{j} + x\hat{k}$ in cylindrical coordinates and find f_ρ, f_ϕ and f_z.

b. Express the vector $\vec{f} = 2y\hat{i} - 2\hat{j} + 3x\hat{k}$ in terms of spherical polar coordinates and find f_r, f_θ, f_ϕ.

PART - D

Answer **Two** full questions. **(2×10=20)**

10. a. Form the partial differential equation by eliminating arbitrary function from $z = e^{ax+by} f(ax - by)$.

b. Solve : $\sqrt{p} + \sqrt{q} = x + y$.

(OR)

11. a. Solve : $\left[D^2 + DD' - 6(D')^2\right] z = \cos(2x + y)$.

b. Solve : $z^2(p^2z^2 + q^2) = 1$.

12. a. Find the complete integral of $z^2(p^2 + q^2 + 1) = 1$ by using charpit's method.

b. Solve : $\left[D^2 - DD'\right] z = \sin x \cos 2y$.

(OR)

[P.T.O.

13. a. Solve : $\frac{\partial u}{\partial t} = 4.\frac{\partial^2 u}{\partial x^2}$, subject to the conditions

i. $u(0,t) = 0$, $u(1,t) = 0$ for all t,

ii. $u(x,0) = x - x^2$, $0 \le x \le 1$.

b. Solve : $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, given $u(0,t) = 0$, $u(1,t) = 0$.

$u(x,0) = k(lx - x^2), \left(\frac{\partial u}{\partial t}\right)_{t=0} = 0$.

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VI Semester B.A./B.Sc. Degree Examination, September/October - 2022

MATHEMATICS - VIII

(CBCS Semester Scheme 2018-19 Freshers & Repeaters 2016-17 & Onwards)

Paper : VIII

Time : 3 Hours **Maximum Marks : 70**

Instructions to Candidates:

1. Answer all questions.
2. Non - programmable scientific calculators are allowed.

PART - A

Answer any **Five** questions. **(5×2=10)**

1. a. Find the locus of the point z satisfying $|z+i| \leq 3$.

b. Evalute $\lim_{z \to 2i} \left(1z^4 + 3z^2 - 10i\right)$.

c. Prove that $u = y^3 - 3x^2 y$ is a harmonic function.

d. Show that $f(z) = \cos z$ is analytic.

e. State fundamental theorem of Algebra.

f. Define cross ratio of four points.

g. Find the real root of the equation $x^3 - 9x + 1 = 0$ in (2.9, 3) by bisection method.

h. State Runge - Kutta method of order 4.

PART - B

Answer **Four** full questions. **(4×10=40)**

2. a. Show that $\arg\left(\frac{\bar{z}}{z}\right) = \frac{\pi}{2}$ represents a line through the origin.

b. State and prove sufficient conditons for $f(z) = u + iv$ to be analytic.

(OR)

[P.T.O.

3. a. Prove that $\lim_{z \to 0}\left(\frac{\bar{z}}{z}\right)$ does not exist.

b. Show that $f(z) = \sin z$ is analytic and hence find $f'(z)$.

4. a. If $f(z) = u + iv$ is an analytic function then prove that the curves $u(x, y) = C_1, v(x, y) = C_2$ forms orthogonal families.

b. Find the analytic function whose real part is $u = x^3 - 3xy^2$.

(OR)

5. a. Show that $u = e^x \sin y + x^2 - y^2$ is harmonic and find its harmonic conjugate.

b. If $f(z) = u + iv$ is analytic function of z, then prove that

$$\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2.$$

6. a. If 'C' is a curve with centre 'a' and radius 'r', then show that

i. $\int_C \frac{dz}{z-a} = 2\pi i$.

ii. $\int_C (z-a)^n dz = 0$ if $n \neq -1$

b. State and prove Cauchy's inequality.

(OR)

7. a. if f(z) is analytic inside and on a simple closed curve 'C' and 'a' is a point within 'C' then prove that

$$f^n(a) = \frac{n!}{2\pi i}\int_C \frac{f(z)}{(z-a)^{n+1}} dz.$$

b. Evaluate $\int_C \frac{dz}{z^2 - 4}$ where 'C' is the circle $|z+2| = 1$.

8. a. Prove that, the bilinear transformation preservers the cross ratio of four points.

b. Find the bilinear transformation which maps z = 1, i, -1 into w = i, 0, -i.

(OR)

9. a. Discuss the transformation $w = \cos z$.

b. Show that the transformation $w = \frac{i-z}{i+z}$ maps the x-axis of the z -plane onto a circle $|w| = 1$ and the points in the half plane y > 0 on the points $|w| < 1$.

PART - C

Answer Two full questions. (2×10=20)

10. a. Find the real root of the equation $x^3 - 4x + 1 = 0$ by Regula - falsi method upto three decimal places.

b. Use Netwon - Raphson method to find a real root of the equation $x^3 - 2x - 5 = 0$.

(OR)

11. a. Solve by Gauss - Jacobi method

$$x + y + 54z = 110,$$
$$27x + 6y - z = 85,$$
$$6x + 15y + 2z = 72,$$

upto four iterations.

b. Find the largest eigen value of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ by power method.

12. a. Use Taylor series method to find y at x = 1.1 and x = 1.2 upto 3rd degree given that $\frac{dy}{dx} = x + y$ and y(1) = 0.

b. Using Euler's method in five stages solve $\frac{dy}{dx} = \frac{y - x}{y + x}$ with y(0) = 1 at x = 0.1.

(OR)

[P.T.O.

13. a. Using Euler's modified method find y(0.1), given $\frac{dy}{dx} = x^2 + y$, y(0) =1 taking h = 0.05.

b. Solve $\frac{dy}{dx} = xy$ given y(1) = 2 at x = 1.2 by using Runge - Kutta method.