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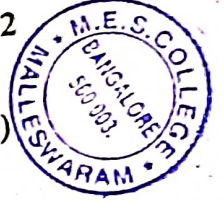
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VI Semester B.A./B.Sc. Degree Examination, September/October - 2022

MATHEMATICS - VII

(CBCS Scheme Semester Freshers & Repeaters 2016-17, 2018-19 And Onwards)

Paper : VII



Time : 3 Hours

Maximum Marks : 70

Instructions to Candidates:

1. Answer all questions.
2. Non - programmable scientific calculators are allowed.

PART - A

Answer any Five questions.

(5×2=10)

1. a. Define a vector space over field.
b. Prove that the set $S = \{(3, 2, -1), (0, 4, 5), (6, 4, -2)\}$ is linearly dependent in $V_3(R)$.
c. Find the matrix of the linear transformation $T: V_2(R) \rightarrow V_2(R)$ defined by $T(x, y) = (x, -y)$ w.r.t the standard bases.
d. Define Rank and Nullity of linear transformation.
e. In a cylindrical coordinate system prove that $\hat{e}_\phi \cdot \hat{e}_z = 0$.
f. Solve $\frac{dx}{yz} = \frac{dy}{zx} = \frac{dz}{xy}$.
g. Form a partial differential equation by eliminating arbitrary constant from $z = (x-a)^2 + (y-b)^2$.
h. Solve $p^2 - q^2 = 1$.

[P.T.O.]



PART - B

Answer two full questions.

(2×10=20)

2. a. Prove that a subset W of a vector space $V(F)$ is a subspace of $V(F)$ if and only if $C_1\alpha + C_2\beta \in W$, for $\alpha, \beta \in W$.
- b. Find the dimension and basis of the subspace spanned by the vectors $(2, -3, 1)$, $(3, 0, 1)$, $(0, 2, 1)$, $(1, 1, 1)$ of $V_3(R)$.

(OR)

3. a. Show that $V = \left\{ \begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix} / x, y \in R \right\}$ is a vector space over R .
- b. Prove that $W = \{(x, y, z) / x = y = z\}$ is a subspace of R^3 .
4. a. Find the linear transformation $T: R^2 \rightarrow R^3$ such that $T(1, 1) = (0, 1, 2)$, $T(-1, 1) = (2, 1, 0)$.
- b. Find the matrix of linear transformation $T: R^2 \rightarrow R^2$ defined by $T(x, y) = (x + 4y, 2x - 3y)$ relative to bases $B_1 = \{(1, 0), (0, 1)\}$, $B_2 = \{(1, 3), (2, 5)\}$.

(OR)

5. a. Let $T: V \rightarrow W$ be a linear transformation, Then show that
- $R(T)$ is a subspace of W .
 - $N(T)$ is a subspace of V .
 - T is one - one if and only if $N(T) = \{0\}$.
- b. Find the range space, null space, rank, nullity and verify Rank - Nullity theorem for $T: V_3(R) \rightarrow V_3(R)$ defined by $T(x, y, z) = (x + y, x - y, 2x + z)$.

PART - C

Answer Two full questions.

(2×10=20)

6. a. Verify the condition of integrability and solve $2yzdx + zxdy - xy(1 + z)dz = 0$.
- b. Solve: $(y^2 + z^2)p - xyq + xz = 0$.

(OR)



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7. a Show that spherical coordinate system is an orthogonal curvilinear coordinate system.
- b Express the vector $\vec{f} = yz\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinates and find f_ρ, f_ϕ and f_z .
8. a Solve $\frac{dx}{1+y} = \frac{dy}{1+x} = \frac{dz}{z}$.
- b Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$.

(OR)

9. a Express the vector $\vec{f} = 3y\hat{i} + 2z\hat{j} + x\hat{k}$ in cylindrical coordinates and find f_ρ, f_ϕ and f_z .
- b Express the vector $\vec{f} = 2y\hat{i} - 2\hat{j} + 3x\hat{k}$ in terms of spherical polar coordinates and find f_x, f_θ, f_ϕ .

PART - D

Answer Two full questions.

(2×10=20)

10. a Form the partial differential equation by eliminating arbitrary function from $z = e^{ax+by} f(ax-by)$.
- b Solve: $\sqrt{p} + \sqrt{q} = x + y$.

(OR)

11. a Solve: $[D^2 + DD' - 6(D')^2]z = \cos(2x + y)$.
- b Solve: $z^2(p^2z^2 + q^2) = 1$.
12. a Find the complete integral of $z^2(p^2 + q^2 + 1) = 1$ by using charpit's method.
- b Solve: $[D^2 - DD']z = \sin x \cos 2y$.

(OR)

[P.T.O.]



13. a. Solve : $\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2}$, subject to the conditions

i. $u(0,t) = 0, u(1,t) = 0$ for all t ,

ii. $u(x,0) = x - x^2, 0 \leq x \leq 1$.

b. Solve : $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, given $u(0,t) = 0, u(1,t) = 0$.

$$u(x,0) = k(lx - x^2), \left(\frac{\partial u}{\partial t} \right)_{t=0} = 0.$$



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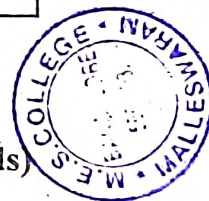
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VI Semester B.A./B.Sc. Degree Examination, September/October - 2022

MATHEMATICS - VIII

(CBCS Semester Scheme 2018-19 Freshers & Repeaters 2016-17 & Onwards)

Paper : VIII



Time : 3 Hours

Maximum Marks : 70

Instructions to Candidates:

1. Answer all questions.
2. Non - programmable scientific calculators are allowed.

PART - AAnswer any **Five** questions.

(5×2=10)

1. a. Find the locus of the point z satisfying $|z+i| \leq 3$.
- b. Evaluate $\lim_{z \rightarrow 2i} (1z^4 + 3z^2 - 10i)$.
- c. Prove that $u = y^3 - 3x^2y$ is a harmonic function.
- d. Show that $f(z) = \cos z$ is analytic.
- e. State fundamental theorem of Algebra.
- f. Define cross ratio of four points.
- g. Find the real root of the equation $x^3 - 9x + 1 = 0$ in $(2.9, 3)$ by bisection method.
- h. State Runge - Kutta method of order 4.

PART - BAnswer **Four** full questions.

(4×10=40)

2. a. Show that $\arg\left(\frac{\bar{z}}{z}\right) = \frac{\pi}{2}$ represents a line through the origin.
- b. State and prove sufficient conditons for $f(z) = u + iv$ to be analytic.

(OR)

[P.T.O.]



3. a. Prove that $\lim_{z \rightarrow 0} \left(\frac{\bar{z}}{z} \right)$ does not exist.
- b. Show that $f(z) = \sin z$ is analytic and hence find $f'(z)$.
4. a. If $f(z) = u + iv$ is an analytic function then prove that the curves $u(x, y) = C_1, v(x, y) = C_2$ forms orthogonal families.
- b. Find the analytic function whose real part is $u = x^3 - 3xy^2$.

(OR)

5. a. Show that $u = e^x \sin y + x^2 - y^2$ is harmonic and find its harmonic conjugate.
- b. If $f(z) = u + iv$ is analytic function of z , then prove that

$$\left(\frac{\partial}{\partial x} |f(z)| \right)^2 + \left(\frac{\partial}{\partial y} |f(z)| \right)^2 = |f'(z)|^2.$$

6. a. If 'C' is a curve with centre 'a' and radius 'r', then show that

i. $\int_C \frac{dz}{z-a} = 2\pi i$.

ii. $\int_C (z-a)^n dz = 0$ if $n \neq -1$

- b. State and prove Cauchy's inequality.

(OR)

7. a. if $f(z)$ is analytic inside and on a simple closed curve 'C' and 'a' is a point within 'C' then prove that

$$f^n(a) = \frac{n!}{2\pi i} \int_C \frac{f(z)}{(z-a)^{n+1}} dz.$$

- b. Evaluate $\int_C \frac{dz}{z^2 - 4}$ where 'C' is the circle $|z+2|=1$.



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8. a. Prove that, the bilinear transformation preserves the cross ratio of four points.
b. Find the bilinear transformation which maps $z = 1, i, -1$ into $w = i, 0, -i$.

(OR)

9. a. Discuss the transformation $w = \cos z$.
b. Show that the transformation $w = \frac{i-z}{i+z}$ maps the x-axis of the z-plane onto a circle $|w| = 1$ and the points in the half plane $y > 0$ on the points $|w| < 1$.

PART - C

Answer Two full questions.

(2×10=20)

10. a. Find the real root of the equation $x^3 - 4x + 1 = 0$ by Regula - falsi method upto three decimal places.
b. Use Newton - Raphson method to find a real root of the equation $x^3 - 2x - 5 = 0$.

(OR)

11. a. Solve by Gauss - Jacobi method

$$x + y + 54z = 110,$$

$$27x + 6y - z = 85,$$

$$6x + 15y + 2z = 72,$$

upto four iterations.

- b. Find the largest eigen value of the matrix $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ by power method.

12. a. Use Taylor series method to find y at $x = 1.1$ and $x = 1.2$ upto 3rd degree given that $\frac{dy}{dx} = x + y$ and $y(1) = 0$.

- b. Using Euler's method in five stages solve $\frac{dy}{dx} = \frac{y-x}{y+x}$ with $y(0) = 1$ at $x = 0.1$.

(OR)

[P.T.O.]



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13. a Using Euler's modified method find $y(0.1)$, given $\frac{dy}{dx} = x^2 + y$, $y(0) = 1$ taking $h = 0.05$.

b. Solve $\frac{dy}{dx} = xy$ given $y(1) = 2$ at $x = 1.2$ by using Runge - Kutta method.
