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GS-323

VI Semester B.A./B.Sc. Examination, May/June - 2019

MATHEMATICS

Mathematics - VII

(CBCS) (F+R) (2016-17 & Onwards)

Time : 3 Hours | Max. Marks : 70

Instructions : *Answer **all** questions.*

PART - A

Answer **any five** sub-questions. **5x2=10**

1. (a) In a vectorspace V(F) show that $C(-\alpha) = -(C\alpha),\ \forall C \in F,\ \alpha \in V$

(b) Prove that the set $S = \{(3, 2, -1), (0, 4, 5), (6, 4, -2)\}$ is Linearly dependent in $V_3(R)$.

(c) Find the matrix of the linear transformation

$T : V_2(R) \to V_2(R)$ defined by

$T(x, y) = (2x+3y,\ 4x-5y)$ with respect to standard bases.

(d) Define Rank and Nullity of linear transformation.

(e) In a cylindrical coordinate system prove that $\hat{e}_\phi \cdot \hat{e}_z = 0$

(f) Solve $\dfrac{x\,dx}{y^2 z} = \dfrac{dy}{zx} = \dfrac{dz}{y^2}$

(g) Form the partial differential equation by eliminating the arbitrary constants from $2Z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$

(h) Solve $\sqrt{p} + \sqrt{q} = 1$

P.T.O.

PART - B

Answer **any two full** questions. **2x10=20**

2. (a) A Subset W of a vectorspace V(F) is a subspace if and only if
   (i) $\alpha, \beta \in W \Rightarrow \alpha + \beta \in W$
   (ii) $C \in F, \alpha \in W \Rightarrow C \cdot \alpha \in W$

   (b) Find the basis and dimension of the subspace spanned by (2, 4, 2), (1, − 1, 0), (1, 2, 1), (0, 3, 1) in $V_3(R)$

OR

3. (a) A set of non zero vectors $(\alpha_1, \alpha_2, \ldots\ldots \alpha_n)$ of vectorspace V(F) is linearly dependent if and only if one of these vectors say α_k $(2 \leq k \leq n)$ is expressed as a linear combination of its preceding ones.

   (b) Show that the subset $W = \{(x_1, x_2, x_3) / x_1 + x_2 + x_3 = 0\}$ is a subspace of $V_3(R)$

4. (a) If $T : U \rightarrow V$ is a linear transformation then prove that.
   (i) $T(0) = 0'$, where 0 and 0' are the zero vectors of U and V respectively.
   (ii) $T(-\alpha) = -T(\alpha), \ \forall \alpha \in U$

   (b) Verify whether $T : V_2(R) \rightarrow V_2(R)$ is a linear transformation defined by $T(x, y) = (3x + 2y, 3x - 4y)$

OR

5. (a) Find the range space, null space, rank, nullity and hence verify rank nullity theorem for $T : V_3(R) \rightarrow V_3(R)$ given by $T(x, y, z) = (x + y, x - y, 2x + z)$

   (b) Show that the linear transformation $T : R^3 \rightarrow R^3$ given by $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 + e_3$, $T(e_3) = e_1 + e_2 + e_3$ is non-singular where $\{e_1, e_2, e_3\}$ is the standard basis of R^3.

PART - C

Answer **any two full** questions. **2x10=20**

6. (a) Verify the condition for integrability and solve $(2x^2 + 2xy + 2xz^2 + 1)\, dx + dy + 2z\, dz = 0$

   (b) Solve $p \tan x + q \tan y = \tan z$

OR

7. (a) Show that the cylindrical coordinate system is Orthogonal Curvilinear Coordinate System.

   (b) Express the vector $\vec{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in cylindrical coordinates and find f_ρ, f_ϕ, f_z

8. (a) Solve $\frac{dx}{x(y^2 - z^2)} = \frac{dy}{y(z^2 - x^2)} = \frac{dz}{z(x^2-y^2)}$

(b) Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$

OR

9. (a) Express the vector $\vec{f} = 3x\hat{i} - 2yz\hat{j} + x^2 z\hat{k}$ in cylindrical coordinates and find f_ρ, f_ϕ, f_z

(b) Express the vector $\vec{f} = x\hat{i} - y\hat{j} + z\hat{k}$ in spherical coordinates and find f_r, f_θ, f_ϕ

PART - D

Answer **any two full** questions. **2x10=20**

10. (a) Form the partial differential equation by eliminating the arbitrary functions $z=f(x+ay)+g(x-ay)$

(b) Solve $p(1+q)=zq$

OR

11. (a) Solve $[D^2-2DD'+(D')^2]z=e^{x+2y}$

(b) Solve $p+q=\sin x+\sin y$

12. (a) Find the complete integral of $px+qy=pq$ by Charpit's method

(b) Solve $[D^2-2DD'+(D')^2]z=12\,xy$

OR

13. (a) A tightly streched string with fixed end points $x=0$ and $x=1$ is initially in a position given by $y = y_0 \sin^3\left(\frac{\pi x}{1}\right)$. If it is released from rest from this position, find the displacement $y(x, t)$.

(b) Solve $\frac{\partial u}{\partial t} = 16\frac{\partial^2 u}{\partial x^2}$ subjected to the conditions.

(i) $u(0, t)=0$, $u(1, t)=0$ for all t

(ii) $u(x, 0)=x^2-x$, $0 \le x \le 1$

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VI Semester B.A./B.Sc. Examination, May/June - 2019

MATHEMATICS

Mathematics - VIII

(CBCS) (Fresh+Repeaters) (2016-17 & Onwards)

Time : 3 Hours

Max. Marks : 70

Instruction : Answer **all** questions.

PART - A

1. Answer **any five** questions. **5x2=10**

(a) Evaluate $\lim_{z\to 1+i} (z^2+2z)$.

(b) Show that $\left|\frac{z-2}{z+2}\right|=3$ represents a circle.

(c) Show that $u=e^x \sin y+x^2-y^2$ is harmonic.

(d) Define cross ratio of four points.

(e) State Liouville's Theorem.

(f) Evaluate $\oint(\bar{z})^2\, dz$ around the circle $|z|=1$

(g) Write Euler modified formula.

(h) State Runge- Kutta Method of order 4.

PART- B

Answer **four** full Questions. **4x10=40**

2. (a) Show that $\arg\left(\frac{z-1+i}{z+1}\right)=\frac{\pi}{4}$

(b) Prove that $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial u}{\partial \theta}=-\frac{\partial v}{\partial r}$

OR

3. (a) Prove that $\lim_{z\to 0}\left(\frac{\bar{z}}{z}\right)$ does not exists.

(b) Show that $f(z)$ = sin z is analytic and hence prove that $f'(z)=\cos z$.

4. (a) Show that $u=y^3-3x^2y$ is harmonic and find its harmonic conjugate.

(b) If $f(z)=u+iv$ is an analytic function then prove that the curves $u(x,y)=c_1$, $v(x,y)=c_2$ form two orthogonal families.

OR

P.T.O.

5. (a) Find the analytic function $f(z)=u+iv$ given that $u-v=e^x(\cos y-\sin y)$.

(b) If $f(z)=u+iv$ is analytic then show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2+\left[\frac{\partial}{\partial y}|f(z)|\right]^2=|f'(z)|^2$

6. (a) Evaluate $\int_{(0,3)}^{(2,4)}[(2y+x^2)dx+(3x-y)dy]$ using the substitution $x=2t$, $y=t^2+3$.

(b) State and Prove Cauchy's Integral Formula.

OR

7. (a) Evaluate $\oint_c \frac{z-4}{z(z^2+9)}dz$ where c is the circle $|z|=1$.

(b) State and prove Cauchy's Integral Theorem.

8. (a) Prove that the Bilinear Transformation preserves the cross ratio of four points.

(b) Discuss the transformation $w=z^2$.

OR

9. (a) Find the Bilinear Transformation which maps $z=1,e^1,-1$ on to $w=i,0,-i$.

(b) Show that the transformation $W=1/z$ transforms a circle to a circle or to a straight line.

PART- C

Answer **two** full questions. **2x10=20**

10. (a) Find the root of the equation $x^3-4x+1=0$ by Regula-falsi method upto three decimal places.

(b) Use Newton-Raphson Method to find a real root of the equation. $x^3-9x+1=0$ near $x=3$.

OR

11. (a) Solve by Guass- Jacobi method.

$10x+2y+z=9$
$x+10y-z=-22$
$-2x+3y+10z=22$

(b) Find the largest Eigen Value of the matrix.

$$A=\begin{pmatrix}1 & 6 & 1\\ 1 & 2 & 0\\ 0 & 0 & 2\end{pmatrix}$$

12. (a) Use Taylor's series method to find y at $x=0.1$ considering terms upto the third degree given $\frac{dy}{dx}=x^2+y^2$ and $y(0)=1$.

(b) Using Euler's modified method, find $y(0.2)$ given $\frac{dy}{dx}=x-y^2$, $y(0)=1$ taking $h=0.1$.

OR

13. (a) Use Euler's Method to solve $\frac{dy}{dx}=x+y$, $y(0)=1$ for $x=0.0\ (0.2)1.0$

(b) Using Runge-Kutta Method find $y(0.2)$ for $\frac{dy}{dx}=x+y$; $y(0)=1$ taking $h=0.2$.

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