VI Semester B.A./B.Sc. Examination, September 2020
(CBCS) (2016 – 17 & Onwards)
(F+R)
Paper – VII : MATHEMATICS

Time : 3 Hours   Max. Marks : 70

Instruction : *Answer all Parts.*

PART – A

1. Answer **any five** questions.   (5×2=10)

   a) In a vector space v over the field F, show that $(-C)\alpha = -(C\alpha)$, $\forall \alpha \in V$, $C \in F$.

   b) Verify whether $w = \{(x_1, x_2, x_3) | x_1^2 + x_2^2 + x_3^2 \leq 0\}$ of $V_3(R)$ is a subspace of $V_3(R)$.

   c) Find the matrix of the linear transformation $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (x, -y)$ w.r.t. the standard bases.

   d) Define Range space and Null space.

   e) Write the scale factors in spherical coordinate system.

   f) Solve : $\dfrac{dx}{z^2 y} = \dfrac{dy}{z^2 x} = \dfrac{dz}{xy^2}$.

   g) Form a partial differential equation by eliminating arbitrary constants from $z = (x-a)^2 + (y-b)^2$.

   h) Solve $p^2 q^3 = 1$.

PART – B

Answer **two full** questions.   (2×10=20)

2. a) Prove that the intersection of any two subspaces of a vector space V[F] is also a subspace of V, but the union of two subspaces of a vector space V[F] need not be a subspace of V.

   b) Find the basis and dimension of the subspace spanned by $\{(1, 2, 3), (3, 1, 0), (-2, 1, 3)\}$ in $V_3(R)$.

                    OR

3. a) Show that the vector (3, –7, 6) is a linear combination of the vectors (1, – 3, 2), (2, 4, 1) and (1, 1, 1).

   b) In an n-dimensional vector space V[F] prove that (i) any (n + 1) elements of V are linearly dependent. (ii) no set of (n – 1) elements can span V.

4. a) Find the linear transformation $T : V_2(R) \rightarrow V_3(R)$ such that T (– 1, 1) = (– 1, 0, 2) and T(2, 1) = (1, 2, 1).

   b) Find the matrix of linear transformation $T : V_3(R) \rightarrow V_2(R)$ defined by

   T(x, y, z) = (x + y, y + z) relative to bases B_1 = {(1, 1, 1), (1, 0, 0), (1, 1, 0)} and

   B_2 = {e_1, e_2} of $V_3(R)$ and $V_2(R)$ respectively.

OR

5. a) Find the range space, null space, rank, nullity and verify RanK-Nullity theorem for $T : V_3(R) \rightarrow V_3(R)$ defined by T(x, y, z) = (x + y, x – y, 2x + z)

   b) Show that the linear transformation $T : V_3(R) \rightarrow V_3(R)$ defined by

   $T(e_1) = e_1 + e_2$, $T(e_2) = e_1 – e_2 + e_3$, $T(e_3) = 3e_1 + 4e_3$ is non singular, where (e_1, e_2, e_3) is the standard basis of $V_3(R)$.

PART – C

Answer **two full** questions.   (2×10=20)

6. a) Verify the condition for integrability and solve

   $z^2 dx + (z^2 – 2yz) dy + (2y^2 – yz – zx) dz = 0$.

   b) Solve $(y^2 + z^2) p – xyq + xz = 0$.

OR

7. a) Show that the cylindrical coordinate system is orthogonal curvilinear coordinate system.

   b) Express the vector $\vec{f} = z\hat{i} – 2x\hat{j} + y\hat{k}$ interms of spherical polar coordinates and find f_r, f_θ, f_ϕ.

8. a) Solve : $\dfrac{dx}{\tan x} = \dfrac{dy}{\tan y} = \dfrac{dz}{\tan z}$.

   b) Solve : $\dfrac{dx}{x^2 - yz} = \dfrac{dy}{y^2 - zx} = \dfrac{dz}{z^2 - xy}$.

OR

9. a) Show that the spherical coordinate system is a orthogonal curvilinear coordinate system.

   b) Express the vector $\vec{f} = 3x\hat{i} - 2yz\hat{j} + x^2z\hat{k}$ interms of cylindrical coordinates and find f_ρ, f_ϕ, f_z.

PART – D

Answer **two full** questions.

$(2 \times 10 = 20)$

10. a) Form the partial differential equation given that $f(x + y + z, x^2 - y^2 - z^2) = 0$.

    b) Solve : $\sqrt{p} + \sqrt{q} = x + y$.

OR

11. a) Solve $[2D^2 - DD' - 3(D')^2] z = 5e^{x-y}$.

    b) Solve $z^2(p^2z^2 + q^2) = 1$.

12. a) Find the complete integral of $px + qy = pq$ by using Charpit's method.

    b) Solve $[D^2 - 3DD' + 2(D')^2] z = \cos(x + 2y)$.

OR

13. a) Solve : $\dfrac{\partial u}{\partial t} = C^2 \dfrac{\partial^2 u}{\partial x^2}$ subject to the conditions.

    i) $u(0, t) = 0$, $u(l, t) = 0$, $t \geq 0$. ii) $u(x, 0) = \dfrac{100x}{l}$, $0 \leq x \leq l$.

    b) Solve $\dfrac{\partial^2 u}{\partial t^2} = C^2 \dfrac{\partial^2 u}{\partial x^2}$ given

    i) $u(0, t) = 0$, $u(l, t) = 0$ ii) $u(x, 0) = k(lx - x^2)$, $\left(\dfrac{\partial u}{\partial t}\right)_{t=0} = 0$

VI Semester B.A./B.Sc. Examination, September 2020
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MATHEMATICS (Paper – VIII)

Time : 3 Hours　　　　　　　　　　　　　　　　　Max. Marks : 70

Instruction :　Answer all Parts.

PART – A

1. Answer **any five** questions.　　　　　　　　　　　　　　(5×2=10)

   a) Evaluate $\lim\limits_{z \to -2i} \dfrac{(2z+3)(z-1)}{z^2 - 2z + 4}$.

   b) Show that $f(z) = z^2 + 2z$ is continuous at $1 + i$.

   c) Show that $u = x^3 - 3xy^2$ is harmonic.

   d) Define Bilinear transformation.

   e) Verify Cauchy-Reimann equations for $f(z) = \sin x \cosh y + i \cos x \sinh y$.

   f) State Liouville's theorem.

   g) Find the real root of the equation $x^3 - 2x - 5 = 0$ in $(2, 3)$ with two iterations by Regula-Falsi method.

   h) Write iterative formula for Runge Kutta method of fourth order.

PART – B

Answer **four full** questions.　　　　　　　　　　　　　　(4×10=40)

2. a) Find locus of the point z satisfying $\left|\dfrac{z-1}{z+i}\right| \ge 2$.

   b) State and prove necessary conditions for a function $f(x, y) = u(x, y) + iv(x, y)$ to be analytic.

OR

3. a) Prove that $\lim\limits_{z \to 0}\left(\dfrac{\bar{z}}{z}\right)$ does not exist.

   b) Show that $f(z) = \sin z$ is analytic and hence find $f'(z)$.

P.T.O.

4. a) Find the analytic function $f(z) = u + iv$ given that $u = e^x(x\cos y - y\sin y)$.

   b) If $f(z) = u + iv$ is an analytic function in the domain of a complex plane, then prove that $u(x, y) = C_1$ and $v(x, y) = C_2$ are orthogonal families where C_1 and C_2 are constants.

OR

5. a) If $f(z) = u + iv$ is analytic, then show that $\left[\dfrac{\partial}{\partial x}|f(z)|\right]^2 + \left[\dfrac{\partial}{\partial y}|f(z)|\right]^2 = |f'(z)|^2$.

   b) Show that $u = x^2 - y^2 + x + 1$ is harmonic function and find its harmonic conjugate.

6. a) Evaluate $\displaystyle\int_{(0,3)}^{(2,4)} (2y + x^2)dx + (3x - y)dy$, along $x = 2t$, $y = t^2 + 3$.

   b) If $f(z)$ is analytic inside and on a simple closed curve 'C' and if 'a' is a point within 'C', then prove that $f(a) = \dfrac{1}{2\pi i}\displaystyle\int_C \dfrac{f(z)}{z - a}dz$.

OR

7. a) Evaluate $\displaystyle\int_C \dfrac{1}{z(z-1)}dz$, where C is the circle $|z| = 3$.

   b) State and prove Cauchys integral theorem.

8. a) Show that the bilinear transformation transforms circles into circles or straight lines.

   b) Discuss the transformation $w = \sin z$.

OR

9. a) Find the bilinear transformation which maps the points $z = 1, i, -1$ to $w = i, 0, -i$.

   b) Prove that the bilinear transformation preserves the cross ratio of four points.

PART – C

Answer **two** full questions.

$(2\times10=20)$

10. a) Use Bijection method in four stages to find a real root of the equation $x^3 - 2x - 5 = 0$.

    b) Using Newton-Raphson method, find the real root of $x^3 + 5x - 11 = 0$ near 1 correct to 3 decimal places.

OR

11. a) Solve by Gauss-Jacobi method,

$x + y + 54z = 110, 27x + 6y - z = 85, 6x + 15y + 2z = 72$

b) Find the largest eigen value of $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ by power method.

12. a) Use Taylor's series method to find y(1.1) considering the terms up to third degree. Given that $\dfrac{dy}{dx} = x + y$, y(1) = 0.

b) Using modified Euler's method to compute y(0.1) given $\dfrac{dy}{dx} = x^2 + y$, y(0) = 1 taking h = 0.05.

OR

13. a) Using modified Euler's method find y(0.1), given $\dfrac{dy}{dx} = x^2 + 1$, y(0) = 1.

b) Using Runge-Kutta method, solve $\dfrac{dy}{dx} = 3x + \dfrac{y}{2}$ with y(0) = 1, compute y(0.2) taking h = 0.2.

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