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DCMT101

Reg. No. | | | | | | | | |

I Semester B.Sc. Degree Examination, March/April - 2024

MATHEMATICS

Algebra - I and Calculus - I

(NEP Scheme 2021-22 and Onwards)

Paper : I MAT DSC 1.1

Time : 2½ Hours **Maximum Marks :60**

Instructions to Candidates:

Answer all questions.

I. Answer any Six questions **(6×2=12)**

1. Find the value of λ for which the system has a non-trivial solution.

$$7x+4y+3z=0$$
$$x+2y+\lambda z=0$$
$$x+3y+2z=0$$

2. Find the eigen values of the matrix $\begin{bmatrix} 1 & 2 \\ 5 & 4 \end{bmatrix}$

3. Find the nth derivative of $e^{3x}\cos 4x$.

4. If $z=\log(x^2+y^2)$ *find* $\frac{\partial z}{\partial x}$ *and* $\frac{\partial z}{\partial y}$.

5. Define continuity of a function.

6. State Rolle's theorem.

7. Evaluate $\lim\limits_{x\to 0} \frac{x-\sin x}{x^3}$.

8. Show that $f(x,y)=x^3+y^3-3xy+1$ is minimum at the point $(1,1)$.

[P.T.O.

II. Answer any Three questions. **(3×4=12)**

9. Find the rank of the matrix $A=\begin{bmatrix}1 & 1 & 1 & 2\\ 2 & 1 & -3 & -6\\ 3 & -3 & 1 & 2\end{bmatrix}$ by reducing it to normal form.

10. Find λ,μ such that the system of equations $x+y+z=6,\ x+2y+3z=10\ x+2y+\lambda z=\mu$ has.

    i) No-solution

    ii) Unique solution

    iii) Many solutions.

11. Find the eigen values and the corresponding eigen vectors of the matrix. $\begin{bmatrix}4 & 1\\ -1 & 2\end{bmatrix}$

12. Verify the Cayley -Hamilton theorem for the matrix $\begin{bmatrix}1 & 0 & 2\\ 0 & 2 & 1\\ 2 & 0 & 3\end{bmatrix}$

13. By using Cayley - Hamilton theorem, find the inverse of the matrix.

$$\begin{bmatrix}0 & 0 & 1\\ 3 & 1 & 0\\ -2 & 1 & 4\end{bmatrix}$$

III. Answer any Three questions **(3×4=12)**

14. Prove that a function which is continuous in a closed interval attains its bounds in the interval.

15. Discuss the continuity of the function

$$f(x)=\begin{cases}x^2-1, & \text{for } x<1\\ 1-\dfrac{1}{x}, & \text{for } x>1\\ 0 & \text{for } x=1\end{cases}$$

16. Discuss the differentiability of $f(x)=|x|$ at $x=0$

17. Find the n[th] derivative of $\frac{2x-1}{(x-2)(x+1)}$.

18. If $y=a\cos(\log x)+b\sin(\log x)$ show that $x^2 y_{n+2}+(2n+1)x\, y_{n+1}+(n^2+1)y_n=0$

IV. **Answer any Three questions** **(3×4=12)**

19. State and prove Cauchy's mean value theorem.

20. Expand $\tan^{-1}x$ in the powers of $\left(x-\frac{\pi}{4}\right)$ by Taylor's theorem.

21. Expand the function $f(x)=\log(1+x)$ up to the term with x^3 by using Maclaurin's expansion.

22. Evaluate $\lim_{x\to 0}\frac{\tan x - x}{x^2\tan x}$

23. Evaluate $\lim_{x\to 0}\left(\frac{1}{x}\right)^{2\tan x}$

V. **Answer any Three questions** **(3×4=12)**

24. If $z=\sin(ax+y)+\cos(ax-y)$, Prove that $\frac{\partial^2 z}{\partial x^2}=a^2\frac{\partial^2 z}{\partial y^2}$.

25. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, $x\neq y$; show that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$.

26. If $u=x+3y^2-z^3$, $v=2x^2-yz$, $w=2z^2-xy$, $Evaluate$ $\frac{\partial(u,v,w)}{\partial(x,y,z)}$ at $(1,-1,0)$

27. Expand $e^x \cos y$ near the point $\left(1, \frac{\pi}{4}\right)$ by taylor's theorem.

28. Find the extreme values of the function $f(x,y) = 2x^2 - xy + y^2 + 7x$.