443672 **DCMT201**

Reg. No.

II Semester B.Sc. Degree Examination, July/August- 2024

**MATHEMATICS**

**Algebra - II and Calculus - II**

**(NEP Scheme)**

**Time : 2 ½ Hours** **Maximum Marks : 60**

***Instructions to Candidates:***

Answer All the questions.

**I.** Answer any Six questions. **(6×2=12)**

1. The binary operation * on the set of positive rational numbers, $Q^+$ is defined by $a * b = \frac{ab}{2}, \forall a, b \in Q^+$. Find the identity element of $Q^+$.

2. Define normal subgroup of a group.

3. Define quotient group of a group.

4. Show that $f:(G,+) \to (G',\cdot)$, defined by $f(x) = e^x, \forall x \in G$ is a homomorphism.

5. Find the angle between radius. vector and tangent for the curve $r = a\,e^{\theta \cot \alpha}$.

6. Find $\frac{ds}{dx}$ for the curve $ay^2 = x^3$.

7. Evaluate $\int_0^{\pi/2} Sin^8 x\, dx$.

8. Write the formula for surface area of the solid generated by the revolution of the area bounded by the curve $y = f(x)$, about the $x$ - axis and the ordinates are $x = a$ and $x = b$.

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II. Answer any **Three** questions. (3×4=12)

9. Prove that $(Z_7, \otimes_7)$ where $Z_7 = \{1, 2, 3, 4, 5, 6\}$ is an abelian group.

10. Let (G, *) be a group and $a, b \in G$, then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$.

11. Prove that every subgroup of a cyclic group is a cyclic.

12. If H is a subgroup of G, then prove that there exists a one - to - one correspondance between any two right cosets of H in G.

13. State and prove Lagrange's theorem in groups.

III. Answer any **Three** questions. (3×4=12)

14. Prove that a subgroup H of a group G is normal if and only if $g h g^{-1} \in H, \ \forall g \in G$ and $h \in H$.

15. If $f : G \to G'$ be a homomorphism, then prove that $f$(H) is a subgroup of $G'$.

16. Prove that product of two normal subgroup of a group is a subgroup of a group..

17. Let $f : G \to G'$ be a homomorphism of groups, with kernel K then prove that $f$ is one - one iff K = {e} where 'e' is the identity element in G.

18. State and prove Cayley's theorem an groups.

IV. Answer any **Three** questions. (3×4=12)

19. Show that the curves $r = a(1 + \cos\theta)$ and $r = b(1 - \cos\theta)$ cut orthogonally.

20. With usual notation prove that $p = r\sin\phi$ and $\frac{1}{p^2} = \frac{1}{r^2} + \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2$.

21. Find the radius of curvature of the cycloid $x = a(\theta + \sin\theta)$ and $y = a(1 - \cos\theta)$.

22. Find all the asymptotes of the curve $x^3 + 2x^2y + xy^2 - x^2 - xy + 2 = 0$.

23. Find the envelope to the curve $\frac{x}{a} + \frac{y}{b} = 1$ and $a + b = c$, where $a$ and $b$ are parameter.

V. Answer any **Three** questions. **(3×4=12)**

24. Evaluate $\int \cos^n x \, dx$.

25. Find the area of the cardioid $r = a(1+\cos\theta)$.

26. The portion of the parabola $y^2 = 4ax$ cut off by the latus rectum, revolves about the tangent at the vertex. Find its volume of the reel thus generated.

27. Find the surface area of revolution of the astroid. $x^{2/3} + y^{2/3} = a^{2/3}$ about the $x$-axis.

28. Evaluate $\int_0^{\pi} x \sin^4 x \cos^2 x \, dx$.