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DCMT301

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III Semester B.Sc. Degree Examination, March/April - 2024

MATHEMATICS

Ordinary Differential Equations and Real Analysis - I

(NEP Scheme)

Paper : 03



Time : 2½ Hours

Maximum Marks :60

Instructions to Candidates :

Answer all questions.

I. Answer any Six questions.

(6×2=12)

1. Show that the equation $(4x + 3y + 1) dx + (3x + 2y + 1) dy = 0$ is exact.
2. Find the general solution of $(y - px)(p - 1) = p$
3. Find the particular integral of $\frac{d^2 y}{dx^2} + y = \sin 3x$
4. Write the necessary and sufficient condition for the total differential equation $Pdx + Qdy + Rdz = 0$ to be integrable.
5. Define a convergent sequence and give an example.
6. Show that the sequence whose n^{th} term is $(-1)^n$ is not a Cauchy sequence.
7. Test the convergence of the series $1^2 + 2^2 + 3^2 + \dots + n^2$.
8. State the Rabee's test for series of positive terms.

II. Answer any THREE questions.

(3×4=12)

9. Verify for exactness and hence solve $(ax + hy + g)dx + (hx + by + f) dy = 0$
10. Solve $p^2 + p(x + y) + xy = 0$
11. Solve $y = p^2 x^4 - px$.
12. Find the general and singular solution of $(x^2 - 1) p^2 - 2xyp + y^2 - 1 = 0$
13. Find the Orthogonal trajectories of the family of parabolas $y^2 = 4ax$ where 'a' is the parameter.

[P.T.O.]

**III. Answer any Three questions**

(3×4=12)

14. Solve $[D^2+3D+2]y = \cos^2 x$.

15. Solve $4x^2 y'' + 4xy' - y = 4x^2$

16. Solve $\frac{d^2 y}{dx^2} + 9y = \sec 3x$ by the method of variation of parameters.

17. Solve $\frac{dx}{dt} - 7x + y = 0, \frac{dy}{dt} - 2x - 5y = 0$

18. Verify the condition for integrability and solve
 $3x^2(y+z)dx + (z^2 + x^3)dy + (2yz + x^3)dz = 0$

IV. Answer any Three questions.

(3×4=12)

19. Test the convergence of the sequences whose n^{th} term is

i) $\left(\frac{2n^2 + 3n + 5}{n + 3} \right) \sin \frac{\pi}{n}$

ii) $\sqrt{n+1} - \sqrt{n}$

20. If $\lim_{n \rightarrow \infty} a_n = a, \lim_{n \rightarrow \infty} b_n = b$ then Show that $\lim_{n \rightarrow \infty} (a_n + b_n) = a + b$.21. Prove that the sequence $\left\{ \frac{2n-7}{3n+2} \right\}$ is

i) Monotonically increasing and

ii) Converges to $\frac{2}{3}$.22. Discuss the nature of the sequence $\left\{ \left(1 + \frac{1}{n} \right)^n \right\}$ 23. Show that the sequence $\{a_n\}$ where $a_n = \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots + \frac{1}{n!}$ is convergent.



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V. Answer any Three questions.

(3×4=12)

24. Test the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$

25. State and Prove D'Alembert's Ratio test for series of positive terms.

26. Test the convergence of the series $\sum \left(\frac{n}{n+1} \right)^{n^2}$

27. Test the convergence, absolute and conditionally convergence of $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

28. Find the sum to infinity of the series $1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \dots$
