DCST401

Reg. No.

IV Semester B.Sc. Degree Examination, July/August - 2024

STATISTICS

Statistical Inference - I

(NEP Scheme 2020)

Paper : IV

Time : 2½ Hours **Maximum Marks : 60**

Instructions to Candidates:

1. **Answer any Eight sub-divisions from Section A and Three questions from Section B.**
2. **Scientific calculators are allowed.**

SECTION - A

I. **Answer any Eight sub-divisions from the following .** **(8×3=24)**

a) Define scale family of p.d.f's with an example.

b) Obtain an unbiased estimator of λ in Poisson (λ) distribution.

c) Given T is an estimator of a parameter θ, then prove that $MSE(T) \geq V(T)$.

d) Show that $T = \sum_{i=1}^{n} Xi$ is sufficient statistic for θ in Bernoulli B(1, θ) distribution.

e) State the properties of maximum likelihood estimators. (MLEs)

f) Obtain moment estimator of μ in normal $N(\mu, \sigma_0^2)$ distribution.

g) Write a note on :

   i) Fisher information function

   ii) Applications of cramer - Rao in equality.

h) What are simple and composite hypotheses? Give examples for each.

i) Define

   i) Test function

   ii) Randomised and non randomised tests.

j) Explain normal test for single population proportion.

[P.T.O.

SECTION - B

II. Answer any Three questions from the following. **(3×12=36)**

2. a) Write the form of one parameter exponential family. Examine whether the family of normal distribution $N(\mu,\sigma^2)$ belongs to exponential family.

b) If $X=(x_1,x_2,....x_n)$ is a random sample from a probability distribution with mean μ and variance σ^2, then show that sample mean and sample mean square are unbiased estimators of these parameters respectively.

c) Show that in a normal distribution $N(\mu,\sigma^2)$, the sample mean is more efficient than sample median.

(3+5+4)

3. a) Obtain the maximum likelihood estimators of $\alpha \; and \; \beta$ in the probability distribution $f(x;\alpha,\beta)=\beta e^{-\beta(x-\alpha)};\ \begin{matrix}\alpha<x<\infty \\ \alpha,\beta>0\end{matrix}$

b) Obtain minimum variance bound estimator of μ in $N(\mu,\sigma_0^2)$ distribution. Also find minimum variance of the estimator.

(5+7)

4. a) If $C=\{x/0.5\le x\}$ is the critical region of a test based on single observation x for testing $H_0:\theta=1$ against $H_1:\theta=2,\theta$ being the parameter of uniform distribution $U(0,\theta)$.

Compute probabilities of type I and type II errors. Also find power of the test.

b) State NP lemma. Obtain most powerful test of level - α for testing H_0: $P=P_0$ against $H_1:P=P_1\,(P_1>P_0)$ in Bernoulli B (1,P) distribution. Also find the expression for power of the test. **(6+6)**

5. a) Describe normal test for testing $H_0:\mu=\mu_0$ against various alternatives.

b) Describe t - test for testing the equality of two means

$H_0:\mu_1=\mu_2 \; against \; H_1:\mu_1\ne\mu_2$

c) Explain F - test for testing $H_0:\sigma_1^2=\sigma_2^2$ against various alternative hypotheses.

(4+4+4)

6. a) Derive ($(1-\alpha)$ 100% confidence interval for the difference of population means $(\mu_1-\mu_2)$ when

i) Population variances are known.

ii) Population variances unknown.

b) Derive ($(1-\alpha)$ 100% confidence interval for binomial population proportion P.

(8+4)