675329

DCMT503

Reg. No.

V Semester B.Sc. Degree Examination, March/April - 2024

MATHEMATICS

Vector Calculus and Analytical Geometry

(NEP Scheme)

Paper : 5.2

Time : 2½ Hours **Maximum Marks : 60**

Instructions to Candidates :

Answer All questions.

I. Answer any SIX questions. **(6×2=12)**

1. If $\vec{r} = t\hat{i} - t^2\hat{j} + \sin t\,\hat{k}$, *find* $\left|\frac{d\vec{r}}{dt}\right|$ *at* $t = 0$.

2. Define curvature and torsion of a space curve.

3. If $\varphi = x^2 - y^2 + 4z$ Show that $\nabla^2\varphi = 0$.

4. If $\varphi(x, y, z) = x^2 + \sin y + z$ find grad φ at $(0, \pi/2, 1)$.

5. Evaluate $\int_{(0,1)}^{(2,5)} (3x + y)dx + (2y - x)dy$ along the curve $y = x^2 + 1$.

6. State Green's theorem in the plane.

7. Show that the planes $x + 2y - 3z + 4 = 0$ *and* $4x + 7y + 6z + 2 = 0$ are perpendicular.

8. Find the centre and radius of the sphere $4x^2 + 4y^2 + 4z^2 - 16x - 24y + 43 = 0$.

II. Answer any THREE questions. **(3×4=12)**

9. If $\vec{a} = 5t^2\hat{i} + t\,\hat{j} - t^3\hat{k}$; $\vec{b} = \sin t\,\hat{i} - \cos t\,\hat{j}$ find $\frac{d}{dt}\left(\vec{a}.\vec{b}\right)$ *and* $\frac{d}{dt}\left(\vec{a} \times \vec{b}\right)$.

10. Derive an expression for curvature and torsion in terms of derivatives of $\vec{r}$ *w.r.t* the parameter t where $\vec{r} = \vec{r}(t)$ is the equation of the curve.

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11. For the curve $x = a\cos t$; $y = a\sin t$; $z = bt$ show that $k = \frac{a}{a^2+b^2}$; $\tau = \frac{b}{a^2+b^2}$.

12. Express the vector $\bar{f} = 3y\hat{i} + x^2\hat{j} - z^2\hat{k}$ in cylindrical polar coordinates.

13. Express the vector $\bar{f} = z\hat{i} - 2x\hat{j} + y\hat{k}$ in spherical polar Co-ordinates.

III. Answer any THREE questions. **(3×4=12)**

14. Find the directional derivative of $\varphi = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the normal to the surface $x \log z - y^2 = -4$ at the point (-1,2,1).

15. Find the angle between the normals to the surface $xy = z^2$ at the points (1,9,-3) and (-2,-2,2).

16. If n is a non- zero constant. Show that $\nabla^2 r^n = n(n+1)r^{n-2}$. Deduce that when $r \neq 0$, r^n is harmonic iff n = -1.

17. If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$ then show that $r^n\bar{r}$ is irrotational vector for any value of n but solenoidal only when n= -3.

18. Prove that div $(\bar{f} \times \bar{g}) = \bar{g}.\, curl\ \bar{f} - \bar{f}.\, Curl\, \bar{g}$.

IV. Answer any THREE questions. **(3×4=12)**

19. Evaluate $\int_c [x+2y]dx + [4-2x]dy$ around the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ in the counter clock wise.

20. Evaluate $\iint_R (x^2 + y^2)dxdy$. where R is region bounded by the line $x = 1$; $y = 0$ and the parabola $y = x^2$.

21. Evaluate $\int_0^1\int_0^1\int_0^1 e^{x+y+z} dxdydz$

22. Find the volume of the sphere $x^2+y^2+z^2=a^2$ using triple integral.

23. Evaluate using Gauss divergence theorem $\bar{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$. taken over the rectangular parallelepiped $x = 0$; $y = 0$; $z = 0$ and $x=a$; $y=b$; $z = c$.

V. **Answer any THREE questions.** **(3×4=12)**

24. Find the bisector of the acute angle between the planes $2x - y + 2z + 3 = 0$ and $3x-2y+6z+8=0$.

25. Find the image of the point (-3,0,1) in the plane $4x - 3y + 2z = 19$.

26. Find the distance between the lines $\frac{x-2}{3}=\frac{y+1}{0}=\frac{z-3}{-1}$ and $\frac{x+1}{3}=\frac{y-2}{0}=\frac{z+4}{-1}$

27. Find the equations of the sphere which passes through the points (0,0,0) ; (1,0,0); (0,1,0) and (0,0,1).

28. Show that the spheres $x^2+y^2+z^2+2x+4y-6z+11=0$ and $5(x^2+y^2+z^2)-13x+19y-25z+45=0$ cut orthogonally and find their plane of intersection.