563442

DCMT501

Reg. No.

V Semester B.Sc. Degree Examination, February/March - 2024

MATHEMATICS

Real Analysis II - Complex Analysis

(NEP Scheme - CORE)

Paper : 5.1

Time : 2½ Hours **Maximum Marks : 60**

Instructions to Candidates:

Answer all questions.

I. Answer any SIX questions. **(6×2=12)**

1. Evaluate $\int_0^\infty \sqrt{x}\, e^{-x} dx$
2. Define Beta function
3. Prove that $u = y^3 - 3x^2y$ is a harmonic function.
4. Evaluate $\lim_{z\to i} \frac{z^2+1}{z^6+1}$
5. State Liouville's theorem.
6. State Cauchy's Integral theorem.
7. Discuss the transformation translation.
8. Find the fixed points of bilinear transformation $w = \frac{3z-4}{z-1}$

II. Answer any THREE questions. **(3×4=12)**

9. Prove that

   i) $\gamma(n+1) = n\,\gamma(n)$

   ii) $\gamma(n+1) = n!$

   Where n is positive integer.

10. Prove that $\beta(m,n)\beta(m+n.P) = \beta(n,p).\beta(n+p.m)$

[P.T.O.

11. Show that $\int_0^{\pi/2} \sin^p\theta \, d\theta \int_0^{\pi/2} \sin^{p+1}\theta \, d\theta = \frac{\pi}{2(p+1)}$

12. Prove that $\beta(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} dx$

13. Prove that $\int_{-1}^{1} (1+x)^{p-1} (1-x)^{q-1} dx = 2^{p+q-1}\beta(p,q)$

III. Answer any THREE questions. **(3×4=12)**

14. State and prove the necessary condition for a function f(z) to be analytic in Cartesian form.

15. Show that $f(z) = \log z$ is analytic and hence prove that $f'(z) = \frac{1}{z}$.

16. If $f(z) = u + iv$ is analytic, prove that $\left(\frac{\partial}{\partial x}|f(z)|\right)^2 + \left(\frac{\partial}{\partial y}|f(z)|\right)^2 = |f'(z)|^2$

17. Prove that the functions u(x,y) and v(x,y) are harmonic conjugates to each other if and only if they are constant functions.

18. Find the analytic function whose real part is $\left(r + \frac{1}{r}\right)\cos\theta$

IV. Answer any THREE questions. **(3×4=12)**

19. Evaluate $\int_c |z|^2 dz$ where c is the square with vertices (0,0), (1,0), (1,1) and (0,1).

20. Evaluate $\int_0^{1+i} (x^2 - iy)dz$ along $y = x^2$.

21. State and prove Cauchy's integral formula.

22. Evaluate $\int_c \frac{e^{2z}}{z+i\pi} dz$ along c : |z - 1| = 1.

23. Evaluate $\int_c \frac{z^2+z+1}{(z-2)^3} dz$ along $c: |z| = 3$

V. **Answer any THREE questions.** **(3×4=12)**

24. Show that the transformation $w = z^2$ transforms the line parallel to imaginary axis to set of confocal parabolas in w-plane.

25. Show that the transformation $w = \frac{1}{z}$ transforms a circle to a circle or to straight lines.

26. Let w_1, w_2, w_3, w_4 be the images of distinct points z_1, z_2, z_3, z_4 in z - plane. Then prove that $(w_1, w_2, w_3, w_4) = (z_1, z_2, z_3, z_4)$

27. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = o, i, \infty$

28. Show that the transformation $w = \frac{i-z}{i+z}$ maps the x - axis of the z - plane onto a circle $|w| = 1$ and the points in the half plane $y > 0$ on the points $|w| < 1$.