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DCMT501



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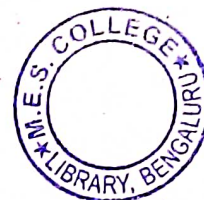
V Semester B.Sc. Degree Examination, February/March - 2024

MATHEMATICS

Real Analysis II - Complex Analysis

(NEP Scheme - CORE)

Paper : 5.1



Time : 2½ Hours

Maximum Marks : 60

Instructions to Candidates:

Answer all questions.

I. Answer any SIX questions.

(6×2=12)

1. Evaluate $\int_0^{\infty} \sqrt{x} e^{-x} dx$
2. Define Beta function
3. Prove that $u = y^3 - 3x^2y$ is a harmonic function.
4. Evaluate $\lim_{z \rightarrow i} \frac{z^2 + 1}{z^6 + 1}$
5. State Liouville's theorem.
6. State Cauchy's Integral theorem.
7. Discuss the transformation translation.
8. Find the fixed points of bilinear transformation $w = \frac{3z-4}{z-1}$

II. Answer any THREE questions.

(3×4=12)

9. Prove that

i) $\gamma(n+1) = n \gamma(n)$

ii) $\gamma(n+1) = n !$

Where n is positive integer.

10. Prove that $\beta(m, n) \beta(m + n, p) = \beta(n, p) \beta(n + p, m)$

[P.T.O.]

11. Show that $\int_0^{\pi/2} \sin^p \theta \, d\theta \int_0^{\pi/2} \sin^{p+1} \theta \, d\theta = \frac{\pi}{2(p+1)}$

12. Prove that $\beta(m, n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}} \, dx$

13. Prove that $\int_{-1}^1 (1+x)^{p-1} (1-x)^{q-1} \, dx = 2^{p+q-1} \beta(p, q)$

III. Answer any THREE questions.

(3×4=12)

14. State and prove the necessary condition for a function $f(z)$ to be analytic in Cartesian form.

15. Show that $f(z) = \log z$ is analytic and hence prove that $f'(z) = \frac{1}{z}$.

16. If $f(z) = u + iv$ is analytic, prove that $\left(\frac{\partial}{\partial x} |f(z)|\right)^2 + \left(\frac{\partial}{\partial y} |f(z)|\right)^2 = |f'(z)|^2$

17. Prove that the functions $u(x, y)$ and $v(x, y)$ are harmonic conjugates to each other if and only if they are constant functions.

18. Find the analytic function whose real part is $\left(r + \frac{1}{r}\right) \cos \theta$

IV. Answer any THREE questions.

(3×4=12)

19. Evaluate $\int_c |z|^2 \, dz$ where c is the square with vertices $(0,0)$, $(1,0)$, $(1,1)$ and $(0,1)$.

20. Evaluate $\int_0^{1+i} (x^2 - iy) \, dz$ along $y = x^2$.

21. State and prove Cauchy's integral formula.

22. Evaluate $\int_c \frac{e^{2z}}{z + i\pi} \, dz$ along $c : |z - 1| = 1$.

23. Evaluate $\int_c \frac{z^2 + z + 1}{(z-2)^3} \, dz$ along $c : |z| = 3$



V. Answer any THREE questions.

(3×4=12)

24. Show that the transformation $w = z^2$ transforms the line parallel to imaginary axis to set of confocal parabolas in w -plane.
25. Show that the transformation $w = \frac{1}{z}$ transforms a circle to a circle or to straight lines.
26. Let w_1, w_2, w_3, w_4 be the images of distinct points z_1, z_2, z_3, z_4 in z -plane. Then prove that $(w_1, w_2, w_3, w_4) = (z_1, z_2, z_3, z_4)$
27. Find the bilinear transformation which maps $z = \infty, i, 0$ into $w = o, i, \infty$
28. Show that the transformation $w = \frac{i-z}{i+z}$ maps the x -axis of the z -plane onto a circle $|w|=1$ and the points in the half plane $y > 0$ on the points $|w| < 1$.
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