



DCMT601

Reg. No.

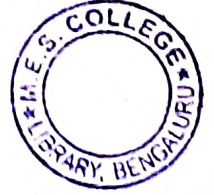
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VI Semester B.Sc. Degree Examination, July/August-2024

MATHEMATICS

Linear Algebra and Calculus of Variations

(NEP Scheme)



Time : 2½ Hours

Maximum Marks : 60

*Instructions to Candidates :*

**Answer All the questions.**

**I. Answer any Six questions.**

(6×2=12)

1. In a ring  $(R, +, \cdot)$ , prove that  $a(-b) = (-a)b = -(a \cdot b) \forall a, b \in R$ .
2. Define a sub ring of a Ring.
3. Prove that in any vector space  $V(F)$ ,  $a \cdot 0 = 0, \forall a \in F$  where '0' be the zero vector of  $V$ .
4. Define Basis and Dimension of a finite dimensional vector space.
5. Verify whether  $T: V_2(R) \rightarrow V_2(R)$  defined by  $T(x, y) = (3x + 2y, 3x - 4y)$  is a linear transformation or not.
6. Define Rank and Nullity of a linear transformation.
7. Write the necessary condition of the euler's equation for the functional to be an extremum.
8. Find the extremal of the functional  $I = \int_{x_1}^{x_2} (1 + xy' + xy'^2) dx$ .

**II. Answer any Three questions.**

(3×4=12)

9. Prove that the set  $S = \{0, 1, 2, 3, 4, 5\}$  is a commutative ring with respect to addition modulo 6 and multiplication modulo 6.
10. Prove that the necessary and sufficient condition for a non-empty subset  $S$  of a ring  $R$  to be a subring of  $R$  are
  - a)  $a \in S, b \in S \Rightarrow a - b \in S$
  - b)  $a \in S, b \in S \Rightarrow ab \in S, \forall a, b \in S$ .

[P.T.O.]



11. Find all the principal ideals of a Ring  $R = \{0, 1, 2, 3, 4, 5, 6, 7\}$  with respect to  $\oplus_8$  and  $\otimes_8$ .
12. If  $f: R \rightarrow R'$  be a homomorphism with kernel  $K$ , then show that  $f$  is one-one if and only if  $K = \{0\}$ .
13. If  $f: R \rightarrow R'$  be an isomorphism of rings  $R$  into  $R'$ , then prove that
  - a) If  $R$  is a commutative ring, then  $R'$  is also a commutative ring.
  - b) If  $R$  is without zero divisors, then  $R'$  is also without zero divisors.

**III. Answer any Three questions.****(3×4=12)**

14. Show that the intersection of any two subspaces of a vector space  $V(F)$  is also a subspace of  $V(F)$ .
15. Prove that a set of non-zero vectors  $\{\alpha_1, \alpha_2, \dots, \alpha_n\}$  of a vector space  $V(F)$  is linearly dependent if and only if one of the vectors  $\alpha_k$  ( $2 \leq k \leq n$ ) is expressed as a linear combination of its preceding ones.
16. Express  $\alpha = (3, 7, -4)$  in  $R^3$  as a linear combination of the vectors  $\alpha_1 = (1, 2, 3)$ ,  $\alpha_2 = (2, 3, 7)$ , and  $\alpha_3 = (3, 5, 6)$ .
17. Show that the set of vectors  $\{(1, 2, 3), (3, 1, 0), (-2, 1, 3)\}$  is not a basis of  $R^3$ . Determine the dimension and basis of the subspace spanned by the given vectors.
18. If  $w_1$  and  $w_2$  are subspaces of a vector space  $V(F)$ , then show that  $w_1 + w_2$  is also a subspace of  $V(F)$ .

**IV. Answer any Three questions.****(3×4=12)**

19. Find a linear transformation  $T: V_2(R) \rightarrow V_3(R)$  such that  $T(-1, 1) = (-1, 0, 2)$ ,  $T(2, 1) = (1, 2, 1)$ .
20. Find the matrix of linear transformation  $T: R^2 \rightarrow R^2$  defined by  $T(x, y) = (x + 4y, 2x - 3y)$  relative to the basis  $B_1 = \{(1, 0), (0, 1)\}$  and  $B_2 = \{(1, 3), (2, 5)\}$ .
21. If  $B_1 = \{\alpha_1, \alpha_2\} = \{(1, 2), (3, 5)\}$  and  $B_2 = \{\beta_1, \beta_2\} = \{(1, -1), (1, -2)\}$  be two bases of  $R^2$ . Find the transition matrix from  $B_1$  to the new basis  $B_2$ .



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22. Find the range space, null space, rank and nullity of the linear transformation  $T: V_3(R) \rightarrow V_2(R)$  defined by  $T(x, y, z) = (y-x, y-z)$ .
23. Find the eigen values and the corresponding eigen vectors for  $T: V_2(R) \rightarrow V_2(R)$  defined by  $T(1, 0) = (1, 2)$ ,  $T(0, 1) = (4, 3)$ .

V. Answer any Three questions.

(3×4=12)

24. Solve the variational problem  $\int_0^{\pi/2} (y^2 - y'^2) dx$  when  $y(0) = 0$ ,  $y\left(\frac{\pi}{2}\right) = 2$ .
25. Show that an extremal of  $\int_{x_1}^{x_2} \left(\frac{y'}{y}\right)^2 dx$  is expressible in the form of  $y = ae^{bx}$ .
26. Prove that the geodesics on a plane are straight lines.
27. If a cable hangs freely under gravity between two fixed points, then show that the shape of the curve is catenary.
28. Show that the extremal of the functional  $\int_0^1 y'^2 dx$  subject to the constraint  $\int_0^1 y dx = 1$  and having  $y(0) = 0$ ,  $y(1) = 1$  is a parabolic arc.
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