DCMT601

Reg. No. | | | | | | | | |

VI Semester B.Sc. Degree Examination, July/August-2024

MATHEMATICS

Linear Algebra and Calculus of Variations

(NEP Scheme)

Time : 2½ Hours **Maximum Marks : 60**

Instructions to Candidates :

Answer All the questions.

I. Answer any Six questions. (6×2=12)

1. In a ring (R, +, .), prove that $a.(-b) = (-a).b = -(a.b)\ \forall a, b \in R$.

2. Define a sub ring of a Ring.

3. Prove that in any vector space V(F), $a.0 = 0$, $\forall a \in F$ where '0' be the zero vector of V.

4. Define Basis and Dimension of a finite dimensional vector space.

5. Verify whether $T : V_2(R) \to V_2(R)$ defined by $T(x, y) = (3x + 2y, 3x - 4y)$ is a linear transformation or not.

6. Define Rank and Nullity of a linear transformation.

7. Write the necessary condition of the euler's equation for the functional to be an extremum.

8. Find the extremal of the functional $I = \int_{x_1}^{x_2} (1 + xy' + xy'^2)\,dx$.

II. Answer any Three questions. (3×4=12)

9. Prove that the set S = {0,1,2,3,4,5} is a commutative ring with respect to addition modulo 6 and multiplication modulo 6.

10. Prove that the necessary and sufficient condition for a non-empty subset S of a ring R to be a subring of R are

 a) $a \in s, b \in s \Rightarrow a - b \in s$

 b) $a \in s, b \in s \Rightarrow ab \in s, \forall a, b \in s$.

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11. Find all the principal ideals of a Ring R={0,1,2,3,4,5,6.7} with respect to $\oplus_8$ *and* $\otimes_8$.

12. If $f: R \to R'$ be a homomorphism with kernel K, then show that f is one-one if and only if K={0}.

13. If $f: R \to R'$ be an isomorphism of rings R into R', then prove that

 a) If R is a commutative ring, then R' is also a commutative ring.

 b) If R is a without zero divisors, then R' is also without zero divisors.

III. Answer any Three questions. **(3×4=12)**

14. Show that the intersection of any two subspaces of a vector space V(F) is also a subspace of V(F).

15. Prove that a set of non-zero vectors $\{\alpha_1, \alpha_2 \ldots\ldots\ldots\ldots \alpha_n\}$ of a vector space V(F) is linearly dependent if and only if one of the vectors α_k $(2 \le k \le n)$ is expressed as a linear combination of its preceeding ones.

16. Express $\alpha = (3,7,-4)$ is R^3 as a linear combination of the vectors $\alpha_1 = (1,2,3),\ \alpha_2 = (2,3,7),\ and\, \alpha_3 = (3,5,6)$.

17. Show that the set of vectors {(1,2,3), (3,1,0), (-2,1,3)} is not a basis of R^3. Determine the dimension and basis of the subspace spanned by the given vectors.

18. If $w_1\ and\ w_2$ are subspaces of a vector space V(F), then show that $w_1 + w_2$ is also a subspace of V(F).

IV. Answer any Three questions. **(3×4=12)**

19. Find a linear transformation $T: V_2(R) \to V_3(R)$ such that T(-1,1)=(– 1,0,2), T(2,1)= (1,2,1).

20. Find the matrix of linear transformation $T: R^2 \to R^2$ defined by $T(x,y) = (x+4y,\ 2x-3y)$ relative to the basis $B_1 = \{(1,0),\ (0,1)\}\ and\ B_2 = \{(1,3),\ (2,5)\}$

21. If $B_1 = \{\alpha_1, \alpha_2\} = \{(1,2),(3,5)\}$ and $B_2 = \{\beta_1, \beta_2\} = \{(1,-1),(1,-2)\}$ be two bases of R^2. Find the transition matrix from B_1 to the new basis B_2.

22. Find the range space, null space, rank and nullity of the linear transformation $T: V_3(R) \to V_2(R)$ defined by $T(x,y,z) = (y-x, y-z)$.

23. Find the eigen values and the corresponding eigen vectors for $T: V_2(R) \to V_2(R)$ defined by $T(1,0) = (1,2)$, $T(0,1) = (4,3)$.

V. **Answer any Three questions.** **(3×4=12)**

24. Solve the variational problem $\int_0^{\pi/2} \left(y^2 - y'^2\right) dx$ when $y(0) = 0$, $y\left(\frac{\pi}{2}\right) = 2$.

25. Show that an extremal of $\int_{x_1}^{x_2} \left(\frac{y'}{y}\right)^2 dx$ is expressible in the form of $y = ae^{bx}$.

26. Prove that the geodesics on a plane are straight lines.

27. If a cable hangs freely under gravity between two fixed points, then show that the shape of the curve is catenary.

28. Show that the extremal of the functional $\int_0^1 y'^2 dx$ subject to the constraint $\int_0^1 y\,dx = 1$ and having $y(0) = 0$, $y(1) = 1$ is a parabolic arc.